Non-renewable resource exploitation: externalities, exploration, scarcity and rents

NRE - Lecture 3

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- Example: pollution
- Effects not covered by markets and so difficult to value
- Trade off between costs and benefits
- $MB = MC$
- Optimal pollution does not imply a fair distribution of costs and benefits!

Externalities *contd.*

- A *social planner* would want a firm to maximise

$$\int_0^T \left[pq - c\left(q, x\right) - d\left(z, a\right) - \theta\left(a\right)\right] e^{-rt} dt$$

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- If $d\left(.\right) \equiv d\left(z - a\right)$ then $d_z = -d_a$ and we can write

$$p - c_q - \theta_a = \lambda$$

for a "socially responsible" firm

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- Note: there may be *stock* as well as *flow* effects

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- Models of exploration are complex!

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- A *trend* in the shadow price is the best indicator of resource scarcity
- Since

$$\dot{p} = \dot{\lambda} + \dot{c}_q,$$

clearly $\dot{p}$ and $\dot{\lambda}$ can have opposite signs

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- A revenue tax is *distortionary*

Rent capture *contd.*

- With a profits tax τ the firm's objective function is

$$\int_0^T [1-\tau]\,[pq - c\,(q,x)]\,e^{-rt}dt$$

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- But, a profits tax reduces the *present value* of the resource
- Government may (also) sell extraction or prospecting rights
- Or, in some cases, the resource itself

Dynamic optimisation: the Hamiltonian

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- Define an instantaneous profit function

$$\pi\left(q\left(t\right),x\left(t\right)\right)\equiv pq\left(t\right)-c\left(q\left(t\right),x\left(t\right)\right)$$

Dynamic optimisation: the Hamiltonian

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- The (social planner's) problem is to

$$\begin{aligned} & \max_{q} \int_{0}^{T} \pi\left(.\right) e^{-rt} dt \\ \text{s.t.} \quad & \dot{x} = f\left(x\left(t\right), q\left(t\right)\right), \quad x\left(0\right) = x_0 \end{aligned}$$

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- The (social planner's) problem is to

$$\max_q \int_0^T \pi(\cdot) e^{-rt} dt$$
$$\text{s.t.} \quad \dot{x} = f(x(t), q(t)), \quad x(0) = x_0$$

- Define a Lagrangian function

$$\mathcal{L} \equiv \int_0^T \left\{ \pi(\cdot) e^{-rt} + \mu(t) \left[f(\cdot) - \dot{x} \right] \right\} dt$$

where

$$\mu(t) \equiv \lambda(t) e^{-rt}$$

Dynamic optimisation *contd.*

- Define a (present value) *Hamiltonian*

$$\mathcal{H}(\centerdot) \equiv \pi(\centerdot)\, e^{-rt} + \mu(t)\, f(\centerdot)$$

so that

$$\mathcal{L} = \int_0^T \mathcal{H}(q, x, \mu, t)\, dt - \int_0^T \mu(t)\, \dot{x} dt$$

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- Integrate the final term *by parts* to obtain

$$\int_0^T \mu(t)\, \dot{x} dt = \mu(T)\, x(T) - \mu(0)\, x_0 - \int_0^T \dot{\mu} x(t)\, dt$$

Dynamic optimisation *contd.*

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- Differentiate w.r.t. q and x to get the FOCs

$$\mathcal{L}_q = \mathcal{H}_q = \pi_q e^{-rt} + \mu f_q = 0$$
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- Hence we require

$$\pi_q e^{-rt} = -\mu f_q \quad \Rightarrow \quad \pi_q e^{-rt} = \mu$$

given $f(\cdot) \equiv g(x) - q$ and hence $f_q = -1$

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- This is equivalent to the familiar *current period* condition

$$\pi_q = \lambda$$

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- Or, without resource-dependent costs,

$$\dot{\lambda} = \lambda r$$

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- The Hamiltonian measures the total rate of increase in the *value* of the resource
  - $\pi\left(.\right)$ is the net flow of returns from the resource
  - $\lambda\left(t\right) f\left(.\right)$ is the increase in the value of the stock