Non-renewable resource exploitation: basic models

NRE - Lecture 2

Aaron Hatcher

Department of Economics
University of Portsmouth

Introduction

- General rule for efficient exploitation of a non-renewable resource

$$v_t'(q_t) = \frac{1}{1+\delta}\lambda_{t+1}, \quad \lambda_{t+1} = \frac{1}{1+\delta}\lambda_{t+2}$$

Introduction

- General rule for efficient exploitation of a non-renewable resource

$$v_t'(q_t) = \frac{1}{1+\delta}\lambda_{t+1}, \quad \lambda_{t+1} = \frac{1}{1+\delta}\lambda_{t+2}$$

- Thus

$$v_t'(q_t) = \frac{1}{1+\delta}v_{t+1}'(q_{t+1})$$

Introduction

- General rule for efficient exploitation of a non-renewable resource

$$v_t'(q_t) = \frac{1}{1+\delta}\lambda_{t+1}, \quad \lambda_{t+1} = \frac{1}{1+\delta}\lambda_{t+2}$$

- Thus

$$v_t'(q_t) = \frac{1}{1+\delta}v_{t+1}'(q_{t+1})$$

- The shadow price is the marginal value of stock left *in situ*

Introduction

- General rule for efficient exploitation of a non-renewable resource

$$v_t'\left(q_t\right) = \frac{1}{1+\delta}\lambda_{t+1}, \quad \lambda_{t+1} = \frac{1}{1+\delta}\lambda_{t+2}$$

- Thus

$$v_t'\left(q_t\right) = \frac{1}{1+\delta}v_{t+1}'\left(q_{t+1}\right)$$

- The shadow price is the marginal value of stock left *in situ*
- In continuous time (without costs)

$$p\left(t\right) = \lambda\left(t\right)$$

and

$$\frac{\dot{p}}{p\left(t\right)} = \frac{\dot{\lambda}}{\lambda\left(t\right)} = r$$

Hotelling's Rule in competitive markets

- With zero extraction costs, we require

$$\dot{p} = rp\left(t\right) > 0$$

Hotelling's Rule in competitive markets

- With zero extraction costs, we require

$$\dot{p} = rp\left(t\right) > 0$$

- How? Downward-sloping demand curve and decreasing supply

Hotelling's Rule in competitive markets

- With zero extraction costs, we require

$$\dot{p} = rp\left(t\right) > 0$$

- How? Downward-sloping demand curve and decreasing supply
- *Arbitrage* maintains the equilibrium rate of price increase

Hotelling's Rule in competitive markets

- With zero extraction costs, we require

$$\dot{p} = rp(t) > 0$$

- How? Downward-sloping demand curve and decreasing supply
- *Arbitrage* maintains the equilibrium rate of price increase
- Recall

$$p(t) = p(T)\, e^{-r[T-t]}$$

Hotelling's Rule in competitive markets

- With zero extraction costs, we require

$$\dot{p} = rp\,(t) > 0$$

- How? Downward-sloping demand curve and decreasing supply
- *Arbitrage* maintains the equilibrium rate of price increase
- Recall

$$p\,(t) = p\,(T)\,e^{-r[T-t]}$$

- The final price $p\,(T)$ is the "backstop" or "choke" price where $q\,(T) = 0$

Hotelling's Rule in competitive markets

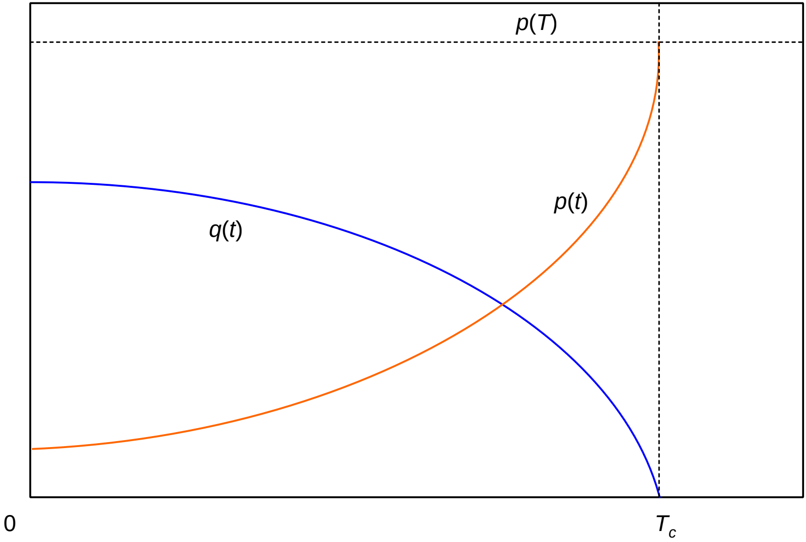
- With zero extraction costs, we require

$$\dot{p} = rp(t) > 0$$

- How? Downward-sloping demand curve and decreasing supply
- *Arbitrage* maintains the equilibrium rate of price increase
- Recall

$$p(t) = p(T)\, e^{-r[T-t]}$$

- The final price $p(T)$ is the "backstop" or "choke" price where $q(T) = 0$
- Without costs, we would expect $x(T) = 0$



p(T)
p(t)
q(t)
0
T_c

Socially optimal extraction

- A *social planner* seeks to maximise total social welfare

Socially optimal extraction

- A *social planner* seeks to maximise total social welfare
- Discounted sum of CS and PS

Socially optimal extraction

- A *social planner* seeks to maximise total social welfare
- Discounted sum of CS and PS
- With zero costs,

$$W\left(q\left(t\right)\right) \equiv \int_{0}^{q\left(t\right)} p\left(q\left(t\right)\right) dq$$

Socially optimal extraction

- A *social planner* seeks to maximise total social welfare
- Discounted sum of CS and PS
- With zero costs,

$$W\left(q\left(t\right)\right) \equiv \int_0^{q\left(t\right)} p\left(q\left(t\right)\right) dq$$

- A competitive firm maximises $\pi\left(t\right) \equiv p\left(t\right) q\left(t\right)$

Socially optimal extraction

- A *social planner* seeks to maximise total social welfare
- Discounted sum of CS and PS
- With zero costs,

$$W(q(t)) \equiv \int_0^{q(t)} p(q(t))\, dq$$

- A competitive firm maximises $\pi(t) \equiv p(t)\, q(t)$
- Since

$$\frac{dW(q(t))}{dq} = \frac{\partial \pi(t)}{\partial q} = p(.)$$

  profit maximisation by competitive firms maximises social welfare

Socially optimal extraction

- A *social planner* seeks to maximise total social welfare
- Discounted sum of CS and PS
- With zero costs,

$$W\left(q\left(t\right)\right) \equiv \int_0^{q(t)} p\left(q\left(t\right)\right) dq$$

- A competitive firm maximises $\pi\left(t\right) \equiv p\left(t\right) q\left(t\right)$
- Since

$$\frac{dW\left(q\left(t\right)\right)}{dq} = \frac{\partial \pi\left(t\right)}{\partial q} = p\left(.\right)$$

  profit maximisation by competitive firms maximises social welfare
- *Assuming* interest rates equal the social discount rate

Extraction by a monopoly

- A monopoly producer maximises

$$\int_0^T p\left(q\left(t\right)\right) q\left(t\right) e^{-rt} dt$$

Extraction by a monopoly

- A monopoly producer maximises

$$\int_0^T p(q(t))\, q(t)\, e^{-rt} dt$$

- But now

$$\frac{d}{dq}\left[p(q(t))\, q(t)\right] = p(.) + \frac{dp(.)}{dq} q(t) \equiv R_q < p(.)$$

Extraction by a monopoly

- A monopoly producer maximises

$$\int_0^T p\left(q\left(t\right)\right) q\left(t\right) e^{-rt} dt$$

- But now

$$\frac{d}{dq}\left[p\left(q\left(t\right)\right) q\left(t\right)\right] = p\left(\cdot\right) + \frac{dp\left(\cdot\right)}{dq} q\left(t\right) \equiv R_q < p\left(\cdot\right)$$

- The monopolist has a downward-sloping marginal revenue curve R_q

Extraction by a monopoly

- A monopoly producer maximises

$$\int_0^T p\left(q\left(t\right)\right) q\left(t\right) e^{-rt} dt$$

- But now

$$\frac{d}{dq}\left[p\left(q\left(t\right)\right) q\left(t\right)\right] = p\left(.\right) + \frac{dp\left(.\right)}{dq} q\left(t\right) \equiv R_q < p\left(.\right)$$

- The monopolist has a downward-sloping marginal revenue curve R_q
- Marginal revenue is less than the market price, *except* at $p\left(T\right)$ where $q\left(T\right) = 0$

Extraction by a monopoly *contd.*

- Hotelling's Rule for a monopoly becomes

$$\frac{\dot{R}_q}{R_q} = r$$

Extraction by a monopoly *contd.*

- Hotelling's Rule for a monopoly becomes

$$\frac{\dot{R}_q}{R_q} = r$$

- The monopolist *controls* the market price

Extraction by a monopoly *contd.*

- Hotelling's Rule for a monopoly becomes

$$\frac{\dot{R}_q}{R_q} = r$$

- The monopolist *controls* the market price
- If discounted marginal revenue is constant, the discounted market price is *decreasing*

Extraction by a monopoly *contd.*

- Hotelling's Rule for a monopoly becomes

$$\frac{\dot{R}_q}{R_q} = r$$

- The monopolist *controls* the market price
- If discounted marginal revenue is constant, the discounted market price is *decreasing*
- The current price is increasing at *less* than the interest rate

Extraction by a monopoly *contd.*

- Hotelling's Rule for a monopoly becomes

$$\frac{\dot{R}_q}{R_q} = r$$

- The monopolist *controls* the market price
- If discounted marginal revenue is constant, the discounted market price is *decreasing*
- The current price is increasing at *less* than the interest rate
- The initial monopoly price is *higher* than the initial competitive price

Extraction by a monopoly *contd.*

- Hotelling's Rule for a monopoly becomes

$$\frac{\dot{R}_q}{R_q} = r$$

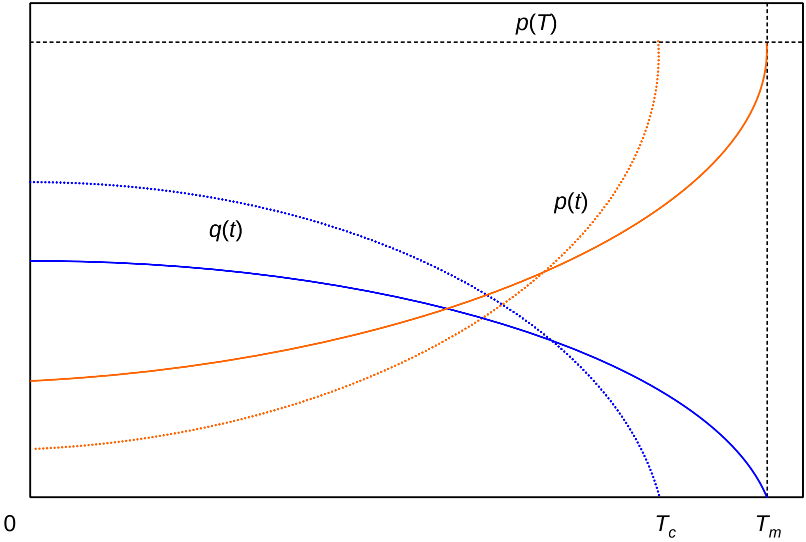
- The monopolist *controls* the market price
- If discounted marginal revenue is constant, the discounted market price is *decreasing*
- The current price is increasing at *less* than the interest rate
- The initial monopoly price is *higher* than the initial competitive price
- The initial quantity extracted is smaller but declines more gradually

Extraction by a monopoly *contd.*

- Hotelling's Rule for a monopoly becomes

$$\frac{\dot{R}_q}{R_q} = r$$

- The monopolist *controls* the market price
- If discounted marginal revenue is constant, the discounted market price is *decreasing*
- The current price is increasing at *less* than the interest rate
- The initial monopoly price is *higher* than the initial competitive price
- The initial quantity extracted is smaller but declines more gradually
- Monopoly extraction is more gradual and extended but not "better" for social welfare



p(T)
p(t)
q(t)
0
T_c
T_m

Costly extraction

- Let firms face a variable cost function

$$c(t) \equiv c(q(t), x(t))$$

Costly extraction

- Let firms face a variable cost function

$$c(t) \equiv c(q(t), x(t))$$

- Here $c_q > 0$ and $c_x \leq 0$

Costly extraction

- Let firms face a variable cost function

$$c(t) \equiv c(q(t), x(t))$$

- Here $c_q > 0$ and $c_x \leq 0$
- Equivalently, $\pi_x \geq 0$

Costly extraction

- Let firms face a variable cost function

$$c\left(t\right) \equiv c\left(q\left(t\right), x\left(t\right)\right)$$

- Here $c_q > 0$ and $c_x \leq 0$
- Equivalently, $\pi_x \geq 0$
- Now efficiency implies

$$\pi_q = \lambda$$

and

$$\dot{\lambda} = \lambda r - \pi_x$$

Costly extraction

- Let firms face a variable cost function

$$c(t) \equiv c(q(t), x(t))$$

- Here $c_q > 0$ and $c_x \leq 0$
- Equivalently, $\pi_x \geq 0$
- Now efficiency implies

$$\pi_q = \lambda$$

  and

$$\dot{\lambda} = \lambda r - \pi_x$$

- Hotelling's Rule for a competitive firm becomes

$$\frac{\dot{\pi}_q}{\pi_q} = r - \frac{\pi_x}{\pi_q}$$

Costly extraction *contd.*

- If $\pi_x > 0$ then
$$\frac{\dot{\pi}_q}{\pi_q} < r$$

Costly extraction *contd.*

- If $\pi_x > 0$ then
$$\frac{\dot{\pi}_q}{\pi_q} < r$$
- Extraction costs moderate the rate of price rise

Costly extraction *contd.*

- If $\pi_x > 0$ then
$$\frac{\dot{\pi}_q}{\pi_q} < r$$
- Extraction costs moderate the rate of price rise
- Otherwise, the efficient extraction path depends on the cost function

Costly extraction *contd.*

- If $\pi_x > 0$ then
$$\frac{\dot{\pi}_q}{\pi_q} < r$$
- Extraction costs moderate the rate of price rise
- Otherwise, the efficient extraction path depends on the cost function
- Extraction may terminate before $x(T) = 0$ and $p(T)$ may not reach the backstop price