Natural resource exploitation: basic concepts

NRE - Lecture 1

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- Renewable resources are capable of *growth* (on some meaningful timescale), e.g., fish, (young growth) forests
- Non-renewable resources are incapable of significant growth, e.g., fossil fuels, ores, diamonds
- In general, efficient and optimal use of natural resources involves *intertemporal* allocation

A capital-theoretic approach

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- This is sometimes called the **short run equation of yield**

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- The value (price) of the resource must increase at a rate equal to the rate of return on the numeraire asset (interest rate)

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- In effect, we require that the *growth rate* of the resource equals the interest rate

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- *Market* interest rates also reflect risk, inflation, taxation, etc.

Discounting and present value

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- Remember that

$$e^{-r} = \frac{1}{1+\delta} \quad \Leftrightarrow \quad r = \ln\left(1+\delta\right)$$

Discounting and present value *contd.*

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- Or in discrete time notation

$$\sum_{t=0}^{T} \left[\frac{1}{1+\delta}\right]^t v_t$$

$$= v_0 + \frac{1}{1+\delta} v_1 + \left[\frac{1}{1+\delta}\right]^2 v_2 + \ldots + \left[\frac{1}{1+\delta}\right]^T v_T$$

A simple two-period resource allocation problem

- The owner of a *non-renewable* resource x_0 seeks to maximise

$$\frac{1}{1+\delta} v_1(q_1) + \left[\frac{1}{1+\delta}\right]^2 v_2(q_2)$$

subject to the constraint

$$q_1 + q_2 = x_0$$

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- The *Lagrangian* function for this problem is

$$\mathcal{L}\equiv\frac{1}{1+\delta}v_1\left(q_1\right)+\left[\frac{1}{1+\delta}\right]^2 v_2\left(q_2\right)+\lambda\left[x_0-q_1-q_2\right]$$

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- The *Lagrangian* function for this problem is

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- The two *first order (necessary) conditions* are

$$\frac{1}{1+\delta} v_1' \left(q_1^* \right) - \lambda = 0, \quad \left[\frac{1}{1+\delta} \right]^2 v_2' \left(q_2^* \right) - \lambda = 0$$

A simple two-period resource allocation problem *contd.*

- Solving the FOCs for the Lagrange multiplier λ we get

$$\frac{v_2'\left(q_2^*\right)}{v_1'\left(q_1^*\right)} = 1 + \delta \quad \Leftrightarrow \quad \frac{v_2'\left(q_2^*\right) - v_1'\left(q_1^*\right)}{v_1'\left(q_1^*\right)} = \delta$$

which is Hotelling's Rule (in discrete time notation)

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- If $v_t\left(q_t\right) \equiv p_t q_t$ (zero extraction costs) then $v_t'\left(q_t\right) = p_t$ and we have

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- In continuous time terms this is equivalent to

$$\frac{\dot{p}}{p\left(t\right)} = r$$

A simple two-period resource allocation problem *contd.*

- Instead, we could attach a multiplier to a stock constraint at each point in time

$$
\begin{aligned}
\mathcal{L} \equiv\ & \frac{1}{1+\delta} v_1\left(q_1\right)+\left[\frac{1}{1+\delta}\right]^2 v_2\left(q_2\right)+\frac{1}{1+\delta} \lambda_1\left[x_0-x_1\right] \\
& +\left[\frac{1}{1+\delta}\right]^2 \lambda_2\left[x_1-q_1-x_2\right]+\left[\frac{1}{1+\delta}\right]^3 \lambda_3\left[x_2-q_2\right]
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& + \left[\frac{1}{1+\delta} \right]^2 \lambda_2 \left[x_1 - q_1 - x_2 \right] + \left[\frac{1}{1+\delta} \right]^3 \lambda_3 \left[x_2 - q_2 \right]
\end{aligned}
$$

- The FOCs for q_1 and q_2 are now

$$
\begin{aligned}
\frac{1}{1+\delta} v_1' \left(q_1^* \right) - \left[\frac{1}{1+\delta} \right]^2 \lambda_2 = 0 \\
\left[\frac{1}{1+\delta} \right]^2 v_2' \left(q_2^* \right) - \left[\frac{1}{1+\delta} \right]^3 \lambda_3 = 0
\end{aligned}
$$

A simple two-period resource allocation problem *contd.*

- If the Lagrangian is maximised by q_1^*, it should also be maximised by x_2^*, so that we can add another FOC

$$-\left[\frac{1}{1+\delta}\right]^2 \lambda_2 + \left[\frac{1}{1+\delta}\right]^3 \lambda_3 = 0$$

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- Substituting for λ_t, we again get

$$v_1'\left(q_1^*\right) = \frac{1}{1+\delta} v_2'\left(q_2^*\right)$$

A simple renewable resource problem

- We can set the problem in terms of a *renewable resource* by incorporating a *growth function* $g_t(x_t)$ into each of the stock constraints

$$\left[\frac{1}{1+\delta}\right]^t \lambda_t \left[x_{t-1} + g_{t-1}(x_{t-1}) - q_{t-1} - x_t\right]$$

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- Solving the Lagrangian as before, we get

$$\frac{1}{1+\delta} v_1'(q_1^*) = \left[\frac{1}{1+\delta}\right]^2 \lambda_2, \quad \left[\frac{1}{1+\delta}\right]^2 v_2'(q_2^*) = \left[\frac{1}{1+\delta}\right]^3 \lambda_3$$

and

$$\left[\frac{1}{1+\delta}\right]^2 \lambda_2 = \left[\frac{1}{1+\delta}\right]^3 \lambda_3 \left[1 + g_2'(x_2^*)\right]$$

A simple renewable resource problem *contd.*

- Solving for λ_t, we now find the intertemporal rule as

$$\frac{v_2'\left(q_2^*\right)}{v_1'\left(q_1^*\right)} = \frac{1+\delta}{1+g_2'\left(x_2^*\right)}$$

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- If $\dot{p} = 0$, we get the yield equation

$$\frac{p \cdot g'\left(x\right)}{p} \equiv \frac{y\left(t\right)}{p\left(t\right)} = r$$