ECON3004 Public Economics- Week 4 Lecture Topic 4: CPRs
This week we covered the topic of Common Pool Resources. Please work on the following question together. A rough sketch of the answer is sufficient.

1. A small seaside village is populated by two individuals, 1 and 2 . The villagers have access to fishing boats, which allows them to source fish for the village. Each villager $h=1,2$ decides on an amount $x_{h}$ of fish to extract, at a cost of $x_{h}\left(x_{1}+\right.$ $x_{2}$ ), i.e., the per unit cost of extraction is increasing in the total amount of fish caught. Villagers derive utility from the fish they extract net of the cost associated with such extraction:

$$
\mathrm{U}_{\mathrm{h}}=\theta \mathrm{x}_{\mathrm{h}}-\mathrm{x}_{\mathrm{h}}\left(\mathrm{x}_{1}+\mathrm{x}_{2}\right) \text {, where } \theta>0 \text { is a preference parameter. }
$$

(a) Derive the optimal level of fish extraction that would maximise the sum of utilities of the two individuals.
(b) Derive the Nash equilibrium if the two individuals simultaneously choose the amounts $\mathrm{x}_{1}$ and $\mathrm{x}_{2}$ of fish to extract non-cooperatively. Compare and contrast your answer to the outcome in (a).
(c) Suppose that the villagers interact repeatedly. How could the villagers use this repeated interaction to achieve the utilitarian outcome?

Hints:

- Take the utility function and re-write it from the perspective of Individual 1, then from the perspective of Individual 2.
- Label each component of the utility function, what does each element represent?
- Look at the cost function, $\mathrm{x}_{\mathrm{h}}\left(\mathrm{x}_{1}+\mathrm{x}_{2}\right)$, how does this change for 1 as 2 catches more fish? Does this make economic sense?
- Set up the optimisation problem that the question requires.
- What welfare approach is the question looking for? What is it called?
- What are you solving for? What is the decision variable that the individual can choose for themselves?
- SOLVE for it! This solution describes the level of the decision variable that maximises the objective function.
- Look at your solution, what does this imply about the individual allocations of fish stocks for 1 and 2?
- Repeat for part (b) and compare with your results form part (a). What can you say about the two? Which one is higher/lower?
- Explain using words (and potentially by illustrating using a payoff matrix?) what repeated interaction can achieve and how this is different from a one-shot game.

