Production process:

input 1 , x_1
input 2 , x_2
⋮
} ⟶ output

machines
labor
aluminium
⋮
} ⟶ cars

Production function: $Q = f(x_1, x_2, \dots)$

Often $Q = f(K, L)$

- then $MPK \equiv \frac{\partial Q}{\partial K} = f_K(K, L)$

$MPL \equiv \partial Q / \partial L = f_L(K, L)$

Example: Cobb-Douglas production function

$$Q = K^a L^b$$

a, b : parameters

$$MPK = \frac{\partial Q}{\partial K} = a K^{a-1} L^b > 0$$

$$MPL = \frac{\partial Q}{\partial L} = b K^a L^{b-1} > 0$$

In macro often $Q = A K^{a} L^{1-a}$

$$MPK = a A K^{a-1} L^{1-a} = a \frac{Q}{K}$$

$$MPL = (1-a) A K^{a} L^{-a} = (1-a) \frac{Q}{L}$$

Consider $\max_{K,L} A K^{a} L^{1-a} - \frac{w}{P} L - \frac{r}{P} K$

$$MPK = \frac{r}{P}$$

$$MPL = \frac{w}{P}$$

Labor share: $\frac{\frac{w}{P} L}{Q} = \frac{(1-a)\frac{Q}{L} L}{Q} = (1-a)$

Capital share: $\frac{\frac{r}{P} K}{Q} = \frac{a \frac{Q}{K} K}{Q} = a$

often use $Q = K^{0.35} L^{0.65}$