Calculating higher-order partial derivatives

Let $f(x,y) = 5x^2y^3$, calculate $\frac{\partial^2 f}{\partial x^2}$, $\frac{\partial^2 f}{\partial x \partial y}$, $\frac{\partial^2 f}{\partial y \partial x}$ and $\frac{\partial^2 f}{\partial y^2}$

Calculate $\frac{\partial^3 f}{\partial x^3}$ and $\frac{\partial^3 f}{\partial y \partial x^2}$.

How many 3rd order partial derivatives of f are there?

If $g(x,y,z) = e^{xy^2} - \ln(x^2 z)$, compute g''_{xx}, g''_{xy} and g''_{yy} at $(4, \frac{1}{2}, \frac{1}{16})$.

Find all 2nd order partial derivatives of $Q = \alpha K^{\frac{1}{4}} L^{3/4}$ where α is a constant.

Calculating higher-order partial derivatives

Let $f(x,y) = 5x^2y^3$, calculate $\frac{\partial^2 f}{\partial x^2}$, $\frac{\partial^2 f}{\partial x \partial y}$, $\frac{\partial^2 f}{\partial y \partial x}$ and $\frac{\partial^2 f}{\partial y^2}$

$$\frac{\partial f}{\partial x} = 10xy^3 \quad , \qquad \frac{\partial f}{\partial y} = 15x^2y^2$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial x}\left(10xy^3\right) = 10y^3,$$

direct

$$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial y}\left(10xy^3\right) = 30xy^2$$

mixed

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial x}\left(15x^2y^2\right) = 30xy^2$$

mixed

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$$

Young's theorem

$$\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial y}\left(15x^2y^2\right) = 30x^2y$$

direct

Calculating higher-order partial derivatives

Let $f(x,y) = 5x^2y^3$, calculate $\frac{\partial^2 f}{\partial x^2}$, $\frac{\partial^2 f}{\partial x \partial y}$, $\frac{\partial^2 f}{\partial y \partial x}$ and $\frac{\partial^2 f}{\partial y^2}$

$$\frac{\partial^2 f}{\partial x^2} = 10y^3, \quad \frac{\partial^2 f}{\partial x \partial y} = 30xy^2 = \frac{\partial^2 f}{\partial y \partial x}, \quad \frac{\partial^2 f}{\partial y^2} = 30x^2y$$

Calculate $\frac{\partial^3 f}{\partial x^3}$ and $\frac{\partial^3 f}{\partial y \partial x^2}$.

$$\frac{\partial^3 f}{\partial x^3} = \frac{\partial}{\partial x}\left(\frac{\partial^2 f}{\partial x^2}\right) = \frac{\partial}{\partial x}\left(10y^3\right) = 0$$

$$\frac{\partial^3 f}{\partial y \partial x^2} = \frac{\partial}{\partial y}\left(\frac{\partial^2 f}{\partial x^2}\right) = \frac{\partial}{\partial y}\left(10y^3\right) = 30y^2$$

2 variables

$2^1 = 2$ 1st order

$2^2 = 4$ 2nd order

$2^3 = 8$ 3rd order

How many 3rd order partial derivatives of f are there?

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x} f\right)\right) = \frac{\partial^3 f}{\partial x^3}, \quad \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x} f\right)\right) = \frac{\partial^3 f}{\partial y^2 \partial x}, \quad \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y} f\right)\right) = \frac{\partial^3 f}{\partial x \partial y^2},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x} f\right)\right) = \frac{\partial^3 f}{\partial y \partial x^2}, \quad \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y} f\right)\right) = \frac{\partial^3 f}{\partial x^2 \partial y}, \quad \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y} f\right)\right) = \frac{\partial^3 f}{\partial y^3}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x} f\right)\right) = \frac{\partial^3 f}{\partial x \partial y \partial x}, \quad \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y} f\right)\right) = \frac{\partial^3 f}{\partial y \partial x \partial y}$$

Calculating higher-order partial derivatives

$-2\ln(x) - \ln(z)$

If $g(x,y,z) = e^{xy^2} - \underline{\ln(x^2 z)}$, compute g''_{xx}, g''_{xy} and g''_{yy} at $(4, \frac{1}{2}, \frac{1}{16})$.

$$g'_x = \frac{\partial g}{\partial x} = y^2 e^{xy^2} - \frac{2}{x}$$

$$g''_{xx} = (g'_x)'_x = y^4 e^{xy^2} + \frac{2}{x^2}$$

$$g''_{xx}(4, \tfrac{1}{2}, \tfrac{1}{16}) = (\tfrac{1}{2})^4 e^{4\times\frac{1}{4}} + \frac{2}{16}$$

$$= \frac{1}{16} e + \frac{2}{16} = \underline{\underline{\frac{e+2}{16}}}$$

$$g''_{xy} = (g'_x)'_y = 2y e^{xy^2} + y^2\, 2xy\, e^{xy^2}$$

$$= 2y e^{xy^2}(1 + xy^2)$$

$$\frac{\partial}{\partial y}\left(\frac{\partial g}{\partial x}\right) = \frac{\partial^2 g}{\partial y \partial x}$$

$$g''_{xy}(4, \tfrac{1}{2}, \tfrac{1}{16}) = 2 \times \tfrac{1}{2} e^1 (1+1) = \underline{\underline{2e}}$$

$$g'_y = 2xy\, e^{xy^2}$$

$$g''_{yy} = (g'_y)'_y = 2x e^{xy^2} + 2xy(2xy\, e^{xy^2})$$

$$= 2x e^{xy^2}(1 + 2xy^2)$$

$$g''_{yy}(4, \tfrac{1}{2}, \tfrac{1}{16}) = 2 \times 4 e^1 (1 + 2)$$

$$= \underline{\underline{24e}}$$

Calculating higher-order partial derivatives

Find all 2nd order partial derivatives of $Q = \alpha K^{\frac{1}{4}} L^{3/4}$ where α is a constant.

$Q_K = \alpha \frac{1}{4} K^{-3/4} L^{3/4}$, $\quad Q_L = \alpha K^{\frac{1}{4}} \frac{3}{4} L^{-1/4}$

$$Q_{KK} = \alpha \frac{1}{4}\left(-\frac{3}{4}\right) K^{-7/4} L^{3/4}$$
$$= -\frac{3\alpha}{16} K^{-7/4} L^{3/4}$$

$$Q_{LL} = \alpha K^{1/4} \frac{3}{4}\left(-\frac{1}{4} L^{-5/4}\right)$$
$$= -\frac{3\alpha}{16} K^{1/4} L^{-5/4}$$

$$Q_{KL} = \alpha \frac{1}{4} K^{-3/4} \frac{3}{4} L^{-1/4}$$
$$= \frac{3\alpha}{16} K^{-3/4} L^{-1/4} = Q_{LK}$$