Calculating first-order partial derivatives

Suppose $f(x,y) = 4x^3y^5$, find $\frac{\partial f}{\partial x}(5,-1)$ and $\frac{\partial f}{\partial y}(5,-1)$

Calculate the 1st order partial derivatives of $\pi = 2p^{\frac{1}{2}}q^{3/2} - 5p^2q$

If $g(x_1, x_2, x_3) = e^{x_1^2 x_2} + \ln(x_1 x_3^2)$, find g_1', g_2' and g_3'

Let $f(x,y,z) = 3x^2y^4z^5$, verify that

$$\frac{d}{dt}\left(f(t,t^2,t^3)\right) = f_x'(t,t^2,t^3) + 2tf_y'(t,t^2,t^3) + 3t^2 f_z'(t,t^2,t^3)$$

Calculating first-order partial derivatives

$g(t)$

$\frac{dg}{dt}$

∂

Suppose $f(x,y) = 4x^3y^5$, find $\frac{\partial f}{\partial x}(5,-1)$ and $\frac{\partial f}{\partial y}(5,-1)$

$$\frac{\partial f}{\partial x}(5,-1) = \frac{d}{dx}\left(f(x,-1)\right)\Big|_{x=5} = \frac{d}{dx}\left(4x^3(-1)^5\right)\Big|_{x=5} = 4(3x^2)(-1)^5\Big|_{x=5}$$

$$= 4(3\times5^2)(-1)^5 = \underline{-300}$$

$$\frac{\partial f}{\partial x}(x,y) = \frac{\partial}{\partial x}(\underline{4y^5}\,x^3) = 4y^5\,3x^2$$

treated as a constant

$$\frac{\partial f}{\partial x}(5,-1) = 4(-1)^5\,3(5^2) = -300$$

$$\frac{\partial f}{\partial y}(x,y) = 4x^3\,5y^4 = 20x^3y^4, \quad \text{so} \quad \frac{\partial f}{\partial y}(5,-1) = 20(5^3)(-1)^4$$

$$= \underline{2500}$$

$g(t)$
g'

Calculating first-order partial derivatives

Calculate the 1^{st} order partial derivatives of $\pi = 2p^{\frac{1}{2}}q^{3/2} - 5p^2q$

$$\underbrace{\frac{\partial \pi}{\partial p} = \frac{\partial}{\partial p}(\pi)}_{\text{Leibniz}} = \underbrace{\pi'_p = \overset{\pi(p,q)}{\pi'_1}}_{\text{Lagrange}} = \overset{\pi(q,p)}{\pi_p} = \partial_p \pi$$

$$\frac{\partial \pi}{\partial p}(p,q), \quad \pi'_p(p,q), \quad \left.\frac{\partial \pi}{\partial p}\right|_{(p,q)}$$

π'_p at (p,q)

$$\pi'_p = \cancel{2}\left(\cancel{\tfrac{1}{2}}p^{-1/2}\right)q^{3/2} - 5(2p)q$$
$$= \underline{\frac{1}{\sqrt{p}}\,q^{3/2} - 10pq}$$

$$\pi'_q = 2p^{\frac{1}{2}}\left(\tfrac{3}{2}q^{\frac{1}{2}}\right) - 5p^2$$
$$= \underline{3p^{\frac{1}{2}}q^{\frac{1}{2}} - 5p^2}$$

Calculating first-order partial derivatives

If $g(x_1, x_2, x_3) = e^{x_1^2 x_2} + \overbrace{\ln(x_1 x_3^2)}^{\ln(x_1) + 2\ln(x_3)}$, find g_1', g_2' and g_3'

$$g_1' = \frac{\partial g}{\partial x_1} = \frac{\partial}{\partial x_1}(x_1^2 x_2)\, e^{x_1^2 x_2} + \frac{\frac{\partial}{\partial x_1}(x_1 x_3^2)}{x_1 x_3^2}$$

$$= 2x_1 x_2\, e^{x_1^2 x_2} + \frac{x_3^2}{x_1 x_3^2}$$

$$= \underline{\underline{2x_1 x_2\, e^{x_1^2 x_2} + \frac{1}{x_1}}}$$

$$g_2' = x_1^2 e^{x_1^2 x_2} + 0 = \underline{\underline{x_1^2 e^{x_1^2 x_2}}}$$

$$g_3' = 0 + 0 + \frac{2}{x_3} = \underline{\underline{\frac{2}{x_3}}}$$

$\boxed{\text{Calculating first-order partial derivatives}}$

Let $f(x,y,z) = 3x^2y^4z^5$, verify that

$$6t^{24} \quad + 24t^{24} \quad +45t^{24} \quad = 75t^{24}$$

$$\frac{d}{dt}\left(f(t,t^2,t^3)\right) = f'_x(t,t^2,t^3) + 2tf'_y(t,t^2,t^3) + 3t^2f'_z(t,t^2,t^3)$$

$$f(t,t^2,t^3) = 3t^2(t^2)^4(t^3)^5 = 3t^2t^8t^{15} = 3t^{25}$$

$$\frac{d}{dt}\left(f(t,t^2,t^3)\right) = \frac{d}{dt}\left(3t^{25}\right) = 3\times 25\,t^{24} = \underline{\underline{75\,t^{24}}}$$

$$f'_x(x,y,z) = 3(2x)y^4z^5 = 6xy^4z^5, \quad \text{so } f'_x(t,t^2,t^3) = 6t(t^2)^4(t^3)^5$$
$$= 6t\,t^8\,t^{15} = 6t^{24}$$

$$f'_y(x,y,z) = 3x^2(4y^3)z^5 = 12x^2y^3z^5,$$

so $f'_y(t,t^2,t^3) = 12t^2(t^2)^3(t^3)^5 = 12t^2t^6t^{15} = 12t^{23}$

so $2tf'_y(t,t^2,t^3) = 2t\,12t^{23} = 24t^{24}$

$$f'_z(x,y,z) = 3x^2y^4\,5z^4 = 15x^2y^4z^4, \quad \text{so } f'_z(t,t^2,t^3) = 15t^2(t^2)^4(t^3)^4$$
$$= 15\,t^2t^8t^{12} = 15t^{22}$$

so $3t^2f'_z(t,t^2,t^3) = 3t^2\,15t^{22} = 45t^{24}$