Functions of several variables

Let $f(x,y) = x^2y^3$, calculate $f(3,-2)$, $f(0,73)$, $f(2t,3t)$

Let $g(u,v,w) = \dfrac{u^2 \ln(v)}{\sqrt{w}}$, compute $g(2, e^5, 9)$

Let $\pi = Ak^{1/4}L^{3/4}$, what is π when $(K,L) = (256,81)$?

Draw the level curves of $f(x,y) = 3x+2y$ & $g(x,y) = 3 - x^2 - y^2$.

Sketch $z = xy$

Functions of several variables

$f(x,y) = x^2 y^3$,

2 inputs, 1 output

$g(u,v,w) = \dfrac{u^2 \ln(v)}{\sqrt{w}}$

3 arguments, 1D value

$z = xy$

2 inputs, 1 output

$L(t) = (3t^2, 4t^3)$,

1 input, 2 output

$H(a,b) = \begin{pmatrix} a+b \\ a-b \end{pmatrix}$,

2D input, 2D value

Functions of several variables

Let $f(x,y) = x^2y^3$, calculate $f(3,-2)$, $f(0,73)$, $f(2t,3t)$

$$f(3,-2) \underset{\substack{x=3\\y=-2}}{=} 3^2(-2)^3 = 9\times(-8) = \underline{\underline{-72}}$$

$$f(-2,3) \underset{\substack{x=-2\\y=3}}{=} (-2)^2 3^3 = 4\times 27 = 108$$

$$f(0,73) = 0^2(73)^3 = 0$$

$$f(2t,3t) = (2t)^2(3t)^3$$

$$= 4t^2\, 27t^3$$

$$= \underline{\underline{108\,t^5}}$$

Functions of several variables

Let $g(u,v,w) = \dfrac{u^2 \ln(v)}{\sqrt{w}}$, compute $g(2, e^5, 9)$

$u=2, \quad v=e^5, \quad w=9$

$$g(2, e^5, 9) = \frac{2^2 \ln(e^5)}{\sqrt{9}} = \frac{4 \times 5}{3} = \frac{20}{3}$$

Functions of several variables

Let $\pi = Ak^{\frac{1}{4}}L^{3/4}$, what is π when $(K,L) = (\underbrace{256}_{K}, \underbrace{81}_{L})$?

$$\pi = A(256)^{\frac{1}{4}}(81)^{3/4}$$

$$\pi = A(4^4)^{\frac{1}{4}}(3^4)^{3/4}$$

$$\pi = A \quad 4^1 \quad 3^3$$

$$\pi = A \times 4 \times 27$$

$$\pi = \underline{108A}$$

Functions of several variables

Draw the level curves of $f(x,y) = 3x+2y$

$$\mathcal{L}_k = \{ \underbrace{(x,y)}_{\text{which of these}} \mid \underbrace{f(x,y) = k}_{\text{have this}} \}$$

$3x+2y = k$

$k=0$ $\quad 3x+2y=0 \iff 2y=-3x \iff y=\frac{-3}{2}x$

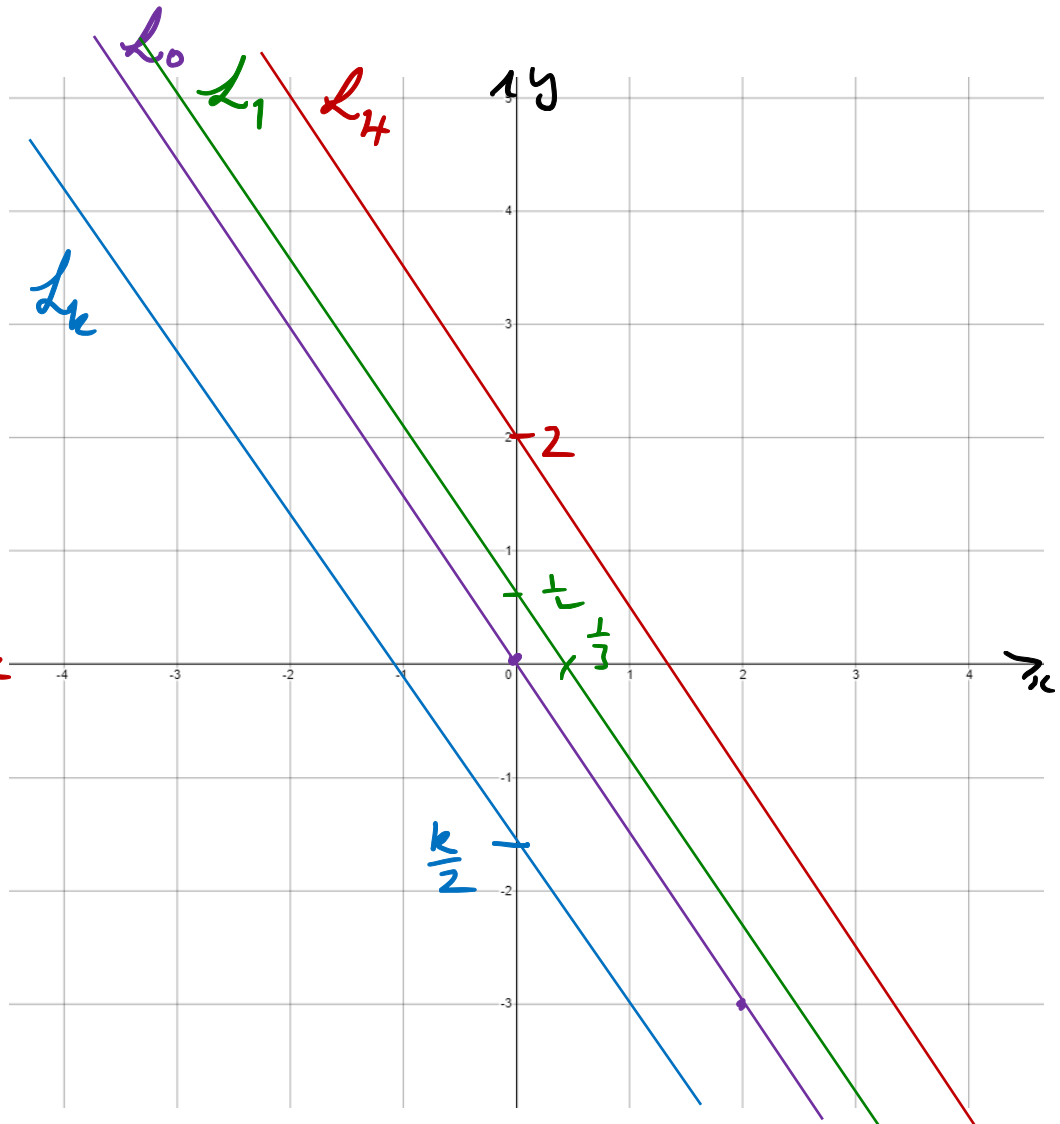
$k=1$ $\quad 3x+2y=1 \iff 2y=1-3x \iff y=\frac{1}{2}-\frac{3}{2}x$

$k=4$ $\quad 3x+2y=4 \iff 2y=4-3x \iff y=2-\frac{3}{2}x$

$3x+2y=k$

$2y=k-3x$

$y=\frac{k}{2}-\frac{3}{2}x$



Functions of several variables

Draw the level curves of $g(x,y) = 3 - x^2 - y^2$.

$3 - x^2 - y^2 = k$

$k=1$ $\quad 3 - x^2 - y^2 = 1$

$2 - x^2 - y^2 = 0$

$x^2 + y^2 = 2$ circle radius $\sqrt{2}$ centered at $(0,0)$

$k=0$ $\quad 3 - x^2 - y^2 = 0$

$x^2 + y^2 = 3$ circle radius $\sqrt{3}$

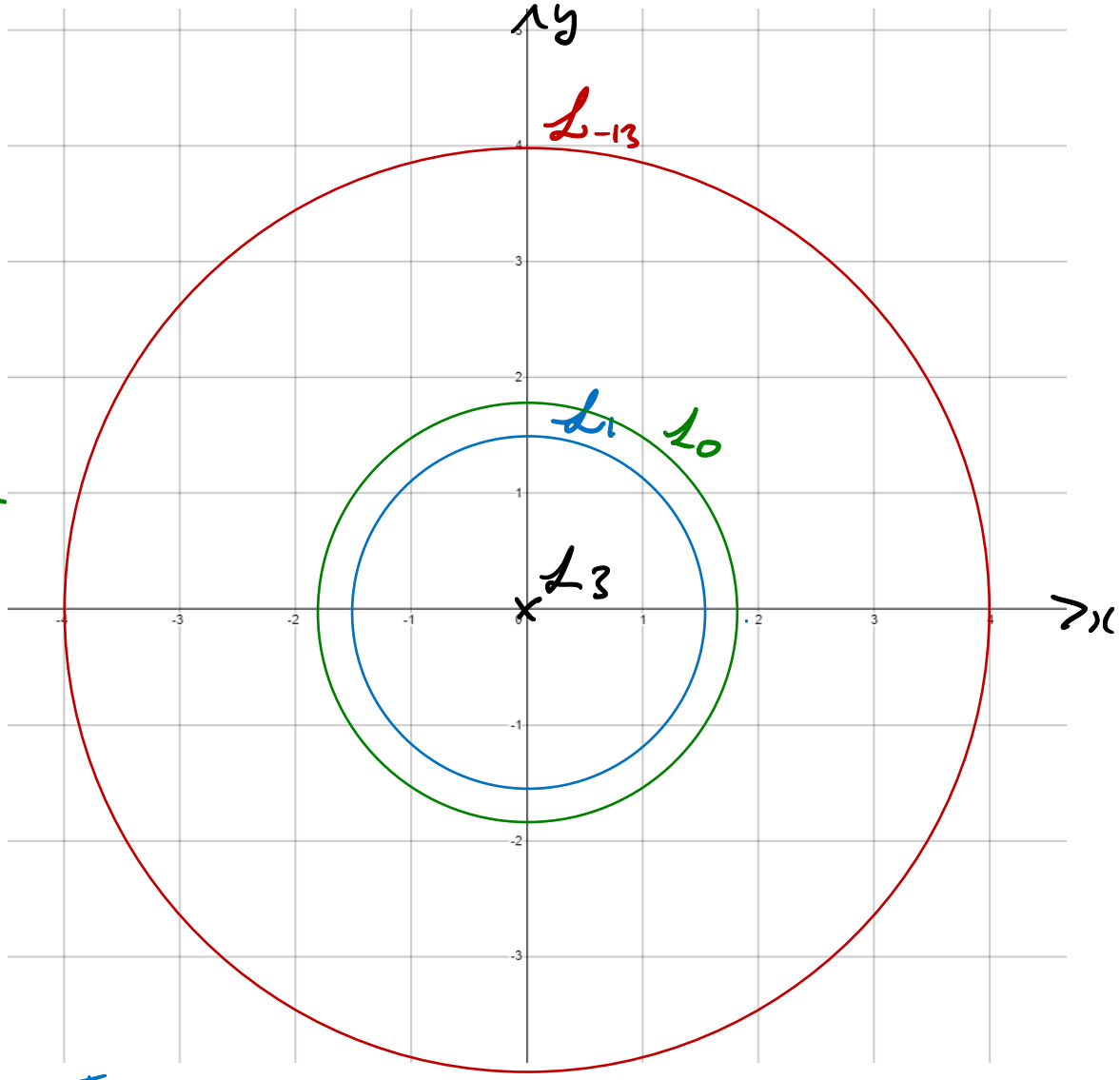
$3 - x^2 - y^2 = k$

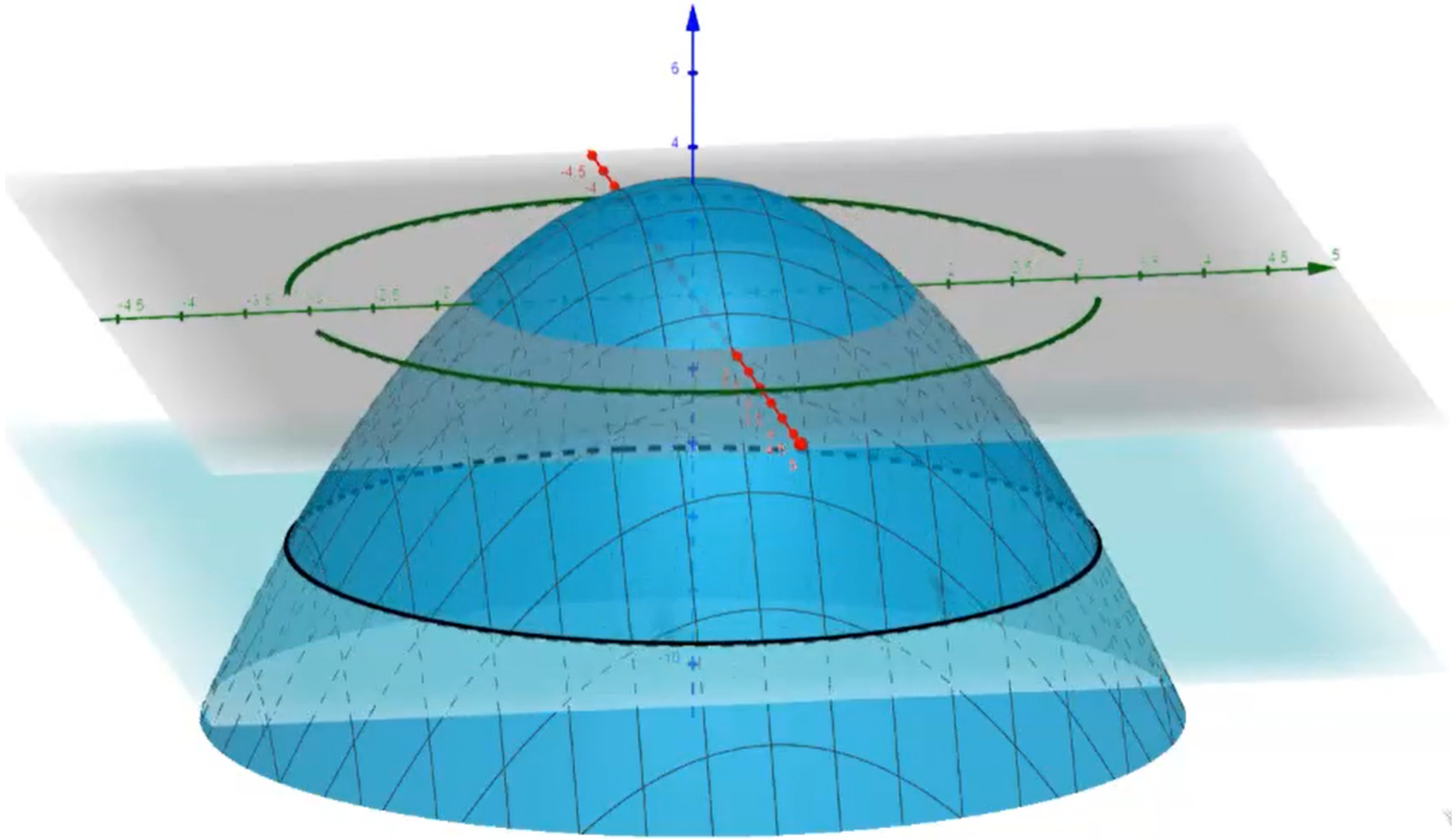
$x^2 + y^2 = 3 - k$

circle radius $\sqrt{3-k}$ if $3-k>0$

$k=3$ $\quad x^2 + y^2 = 0$ $(0,0)$

if $k<3$, $x^2 + y^2 = 3 - k < 0$ no solutions L_k empty




6
4
-4.5
-4
5
-4.5
-4
-10

Functions of several variables

Sketch $z = xy$

level curves $xy = k$

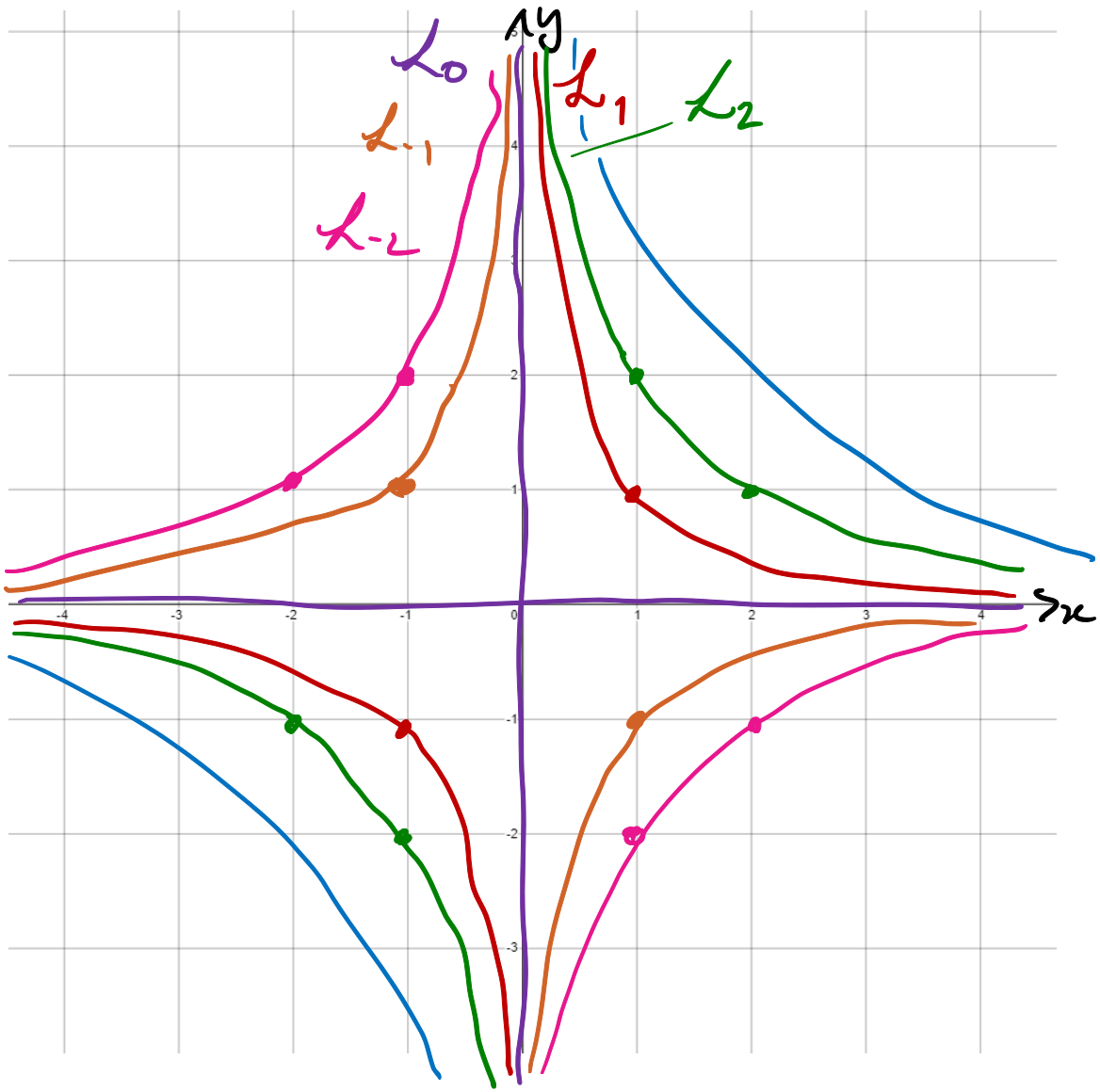
$k=1$ $\quad xy = 1 \iff y = \frac{1}{x}$

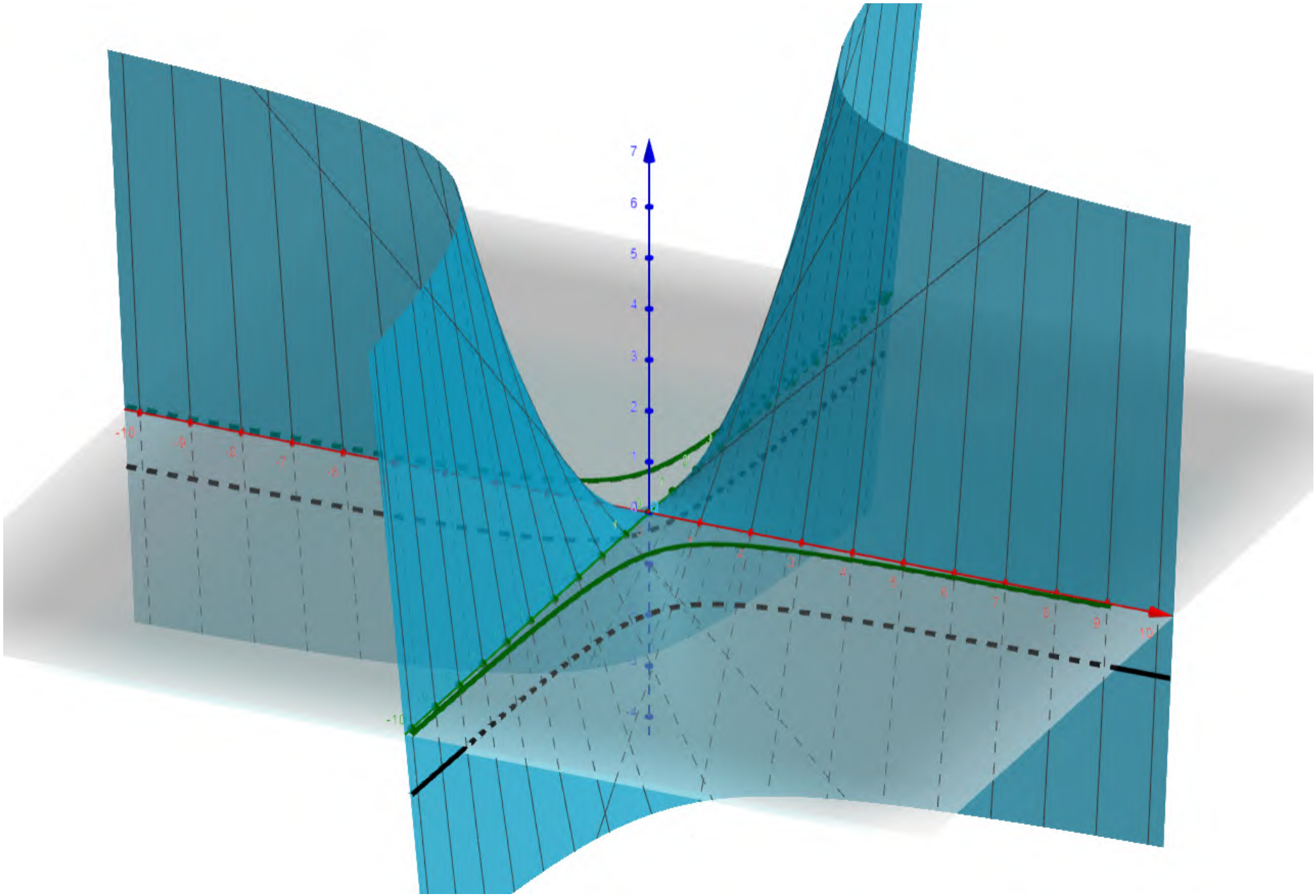
$k=2$ $\quad xy = 2 \iff y = \frac{2}{x}$

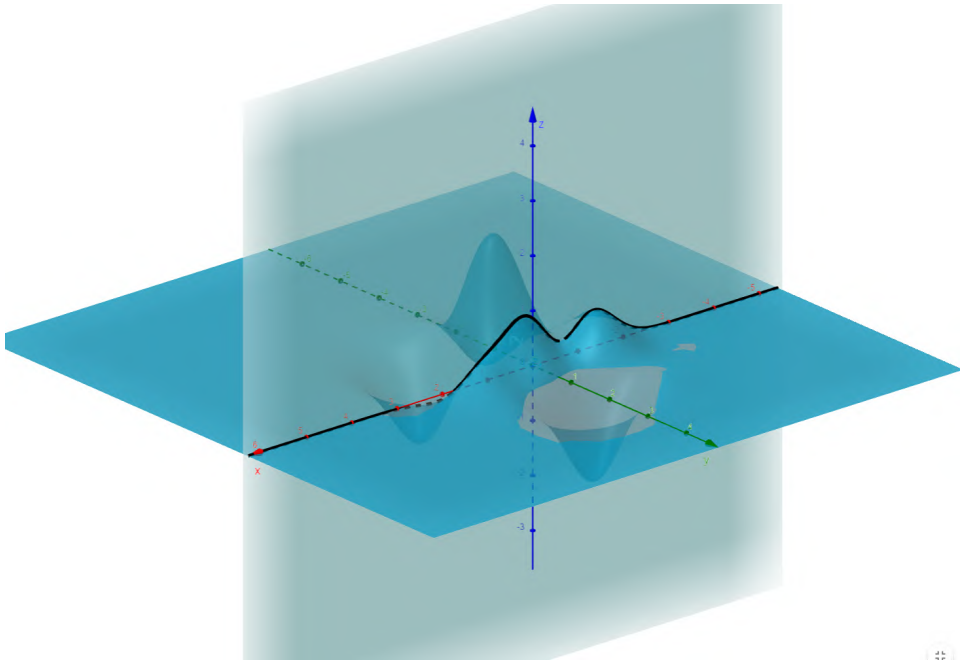
$k=0$ $\quad xy = 0 \iff x = 0$ or $y = 0$

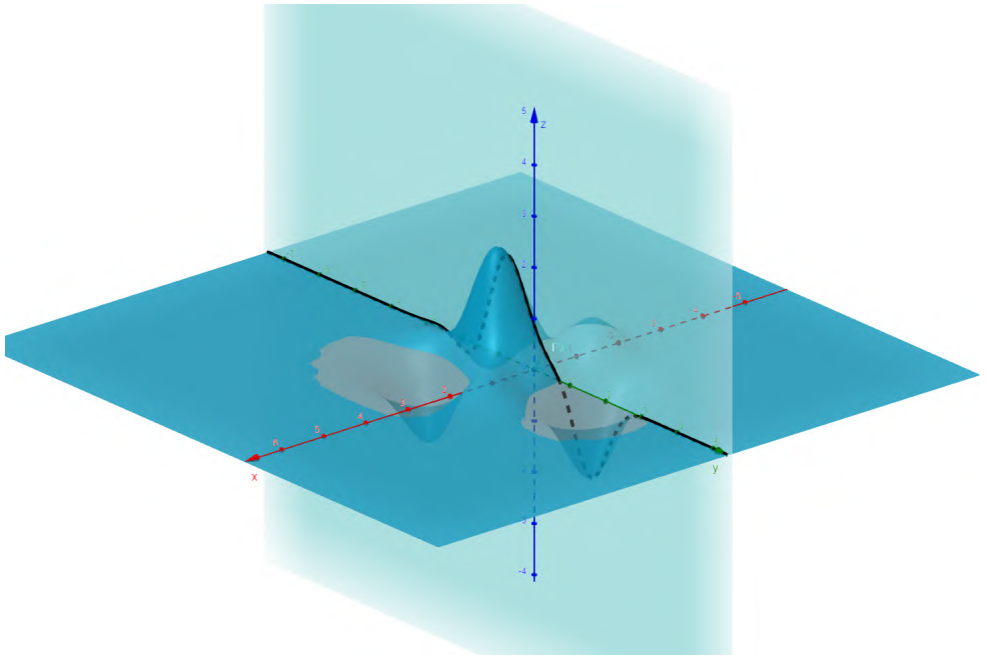
$k=-1$ $\quad xy = -1 \iff y = -1/x$

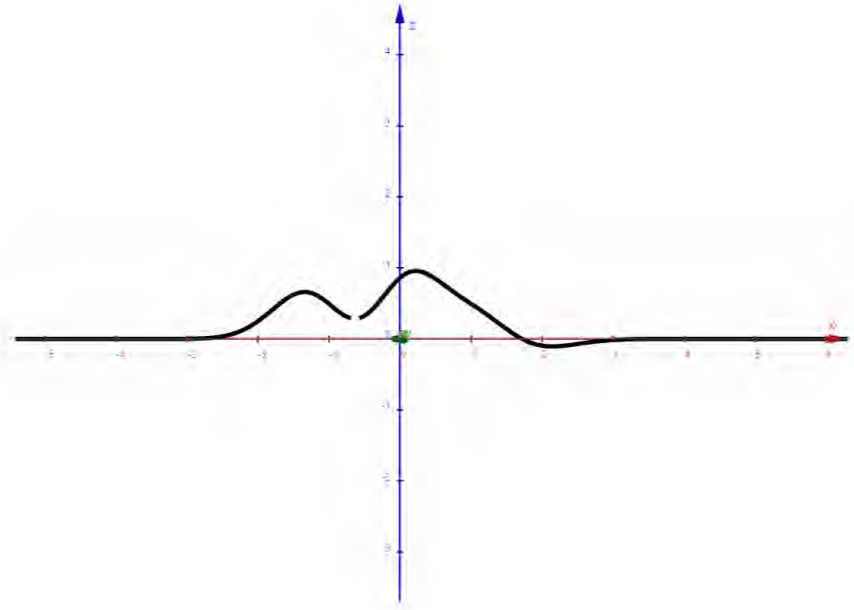
$k=-2$ $\quad xy = -2 \iff y = -\frac{2}{x}$

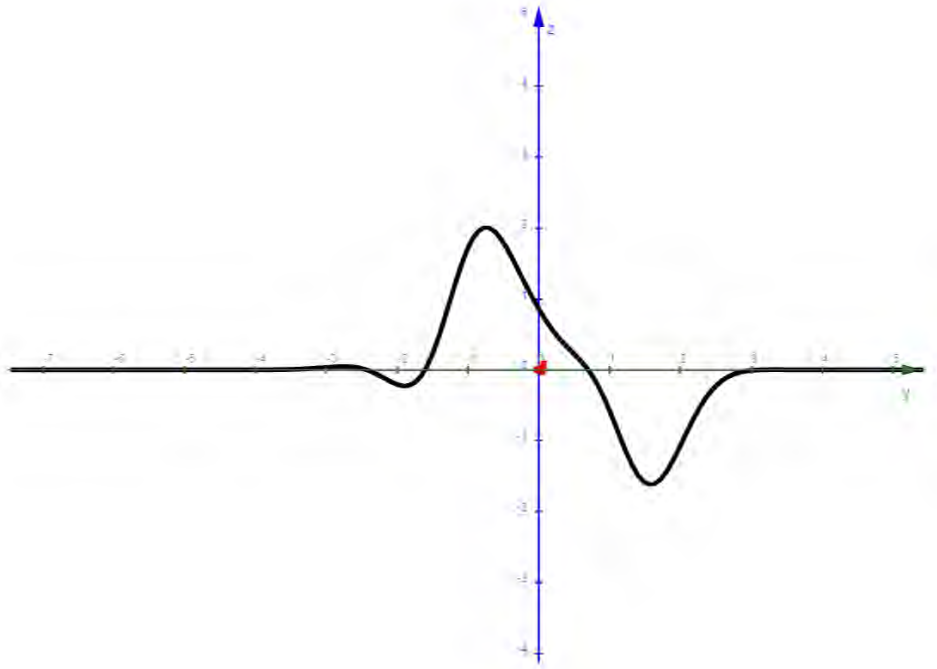



7
6
5
4
3
2
1
-10
10


z
x
y


z
x
y


z
x


z
y