Computing integrals as antiderivatives

If $\frac{dy}{dx} = 7x^5$, and $y(0) = 2$, then what is $y(x)$?

Compute $\int 2y^2 + 5y - 3\,dy$

If $\int g(t)\,dt = 3t^4$, what is $g(t)$?

Calculate $\int 2e^{5z}\,dz$ and $\int \frac{5}{3t+2}\,dt$

Computing integrals as antiderivatives

If $\frac{dy}{dx} = 7x^5$, and $y(0) = 2$, then what is $y(x)$?

$\frac{d}{dx}(x^n) = n x^{n-1}$

$n-1=5$

$n=6$

$\frac{d}{dx}(x^6) = 6x^5$

$\frac{d}{dx}\left(\frac{7}{6}x^6\right) = \frac{7}{6}\frac{d}{dx}(x^6)$

$= \frac{7}{6}6x^5$

$= 7x^5$

$\frac{d}{dx}\left(\frac{7}{6}x^6 + 2\right) = 7x^5$

$\frac{d}{dx}\left(\frac{7}{6}x^6 + C\right) = 7x^5$

if $\frac{dy}{dx} = 7x^5$

then $\frac{d}{dx}\left(y - \frac{7}{6}x^6\right) = \frac{dy}{dx} - \frac{d}{dx}\left(\frac{7}{6}x^6\right)$

$= 7x^5 - 7x^5 = 0$

so $y - \frac{7}{6}x^6 = C$ for some constant

$y = \underbrace{\frac{7}{6}x^6 + C}$ ← constant of integration

antiderivative

$\int 7x^5\,dx$ ← indefinite integral

At $x=0$, $y=2$.

"initial condition" $2 = \frac{7}{6}0^6 + C \quad \therefore C = 2$

$y(x) = \frac{7}{6}x^6 + 2$

Computing integrals as antiderivatives

Compute $\int 2y^2 + 5y - 3\,dy$

$\frac{d}{dy}(y^n) = ny^{n-1}$

$\frac{d}{dy}\left(\frac{1}{n}y^n\right) = y^{n-1}$

$\int y^{n-1}\,dy = \frac{y^n}{n} + C$

$\int 2y^2\,dy = 2\frac{y^3}{3} + C_1$

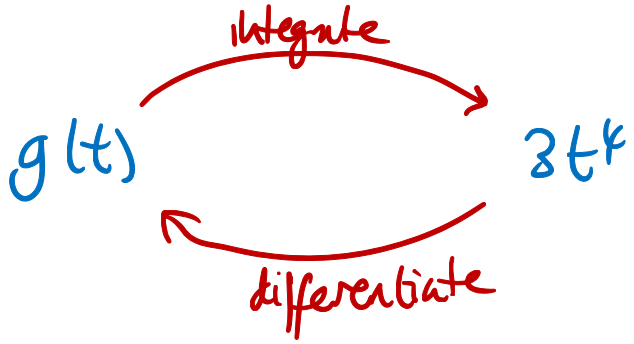
$\int 5y\,dy = 5\frac{y^2}{2} + C_2$

$\int -3\,dy = \int -3y^0\,dy = -3\frac{y^1}{1} + C_3$

$$\int 2y^2 + 5y - 3\,dy = \frac{2}{3}y^3 + \frac{5}{2}y^2 - 3y + C$$

Computing integrals as antiderivatives

If $\int g(t)\,dt = 3t^4$, what is $g(t)$?



$$g(t) = \frac{d}{dt}(3t^4) = \underline{12t^3}$$

Computing integrals as antiderivatives

Calculate $\int 2e^{5z}\,dz = \frac{2}{5}e^{5z} + C$

$\frac{d}{dt}(e^{at}) = ae^{at}$

$\frac{d}{dt}(e^{5t}) = 5e^{5t}$

$\frac{d}{dt}\left(\frac{2}{5}e^{5t}\right) = \frac{2}{5}5e^{5t}$

$= 2e^{5t}$

Computing integrals as antiderivatives

Calculate $\int \frac{5}{3t+2}\,dt$

$$\frac{d}{dt}\left(\ln(at+b)\right) = \frac{a}{at+b}$$

$$\frac{d}{dt}\left(\ln(3t+2)\right) = \frac{3}{3t+2}$$

$$\int \frac{5}{3t+2}\,dt = \int \frac{5}{3}\,\frac{3}{3t+2}\,dt$$

$$= \frac{5}{3}\int \frac{d}{dt}\left(\ln(3t+2)\right)dt$$

$$= \frac{5}{3}\ln(3t+2) + C$$