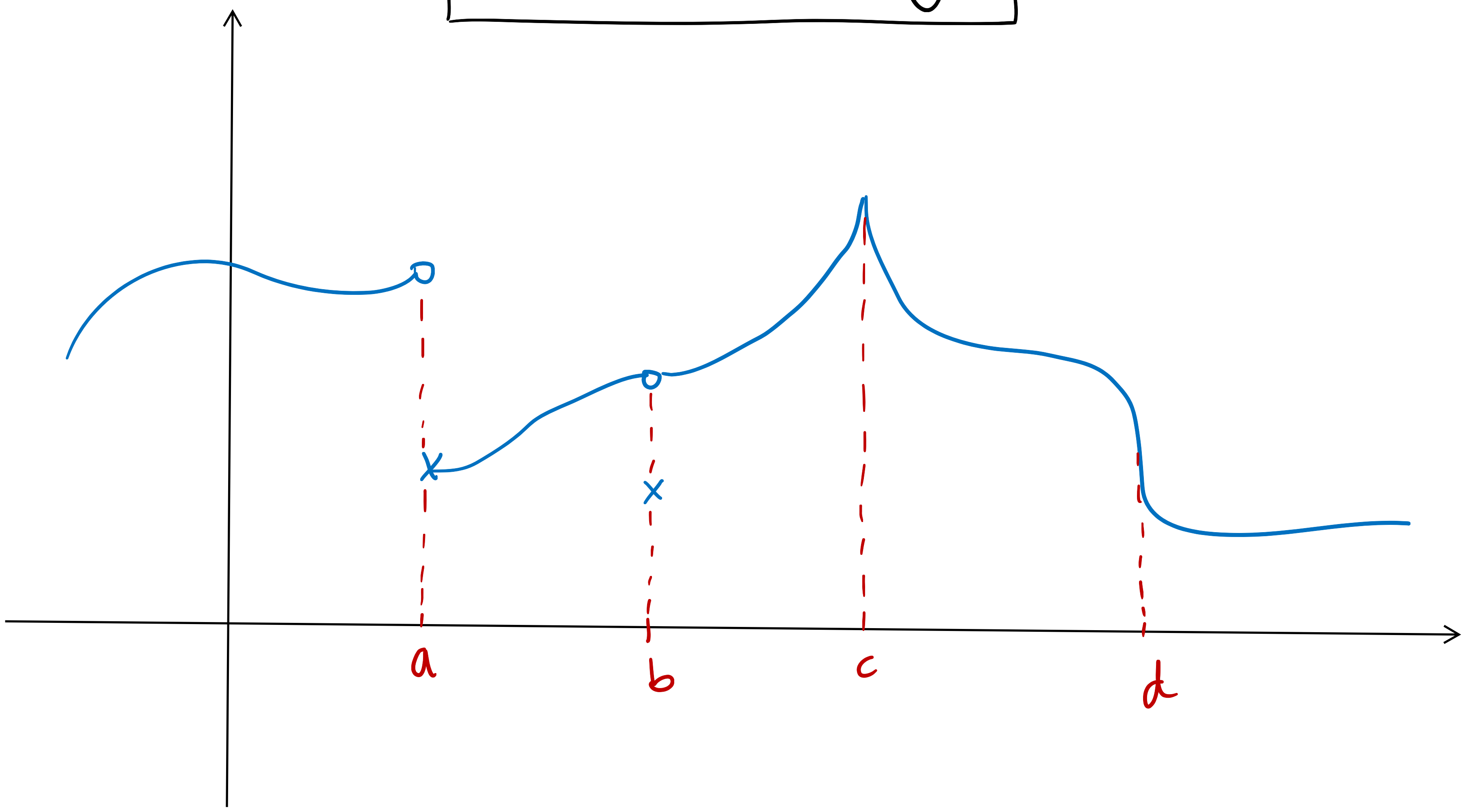
Non-differentiability



Non-differentiability

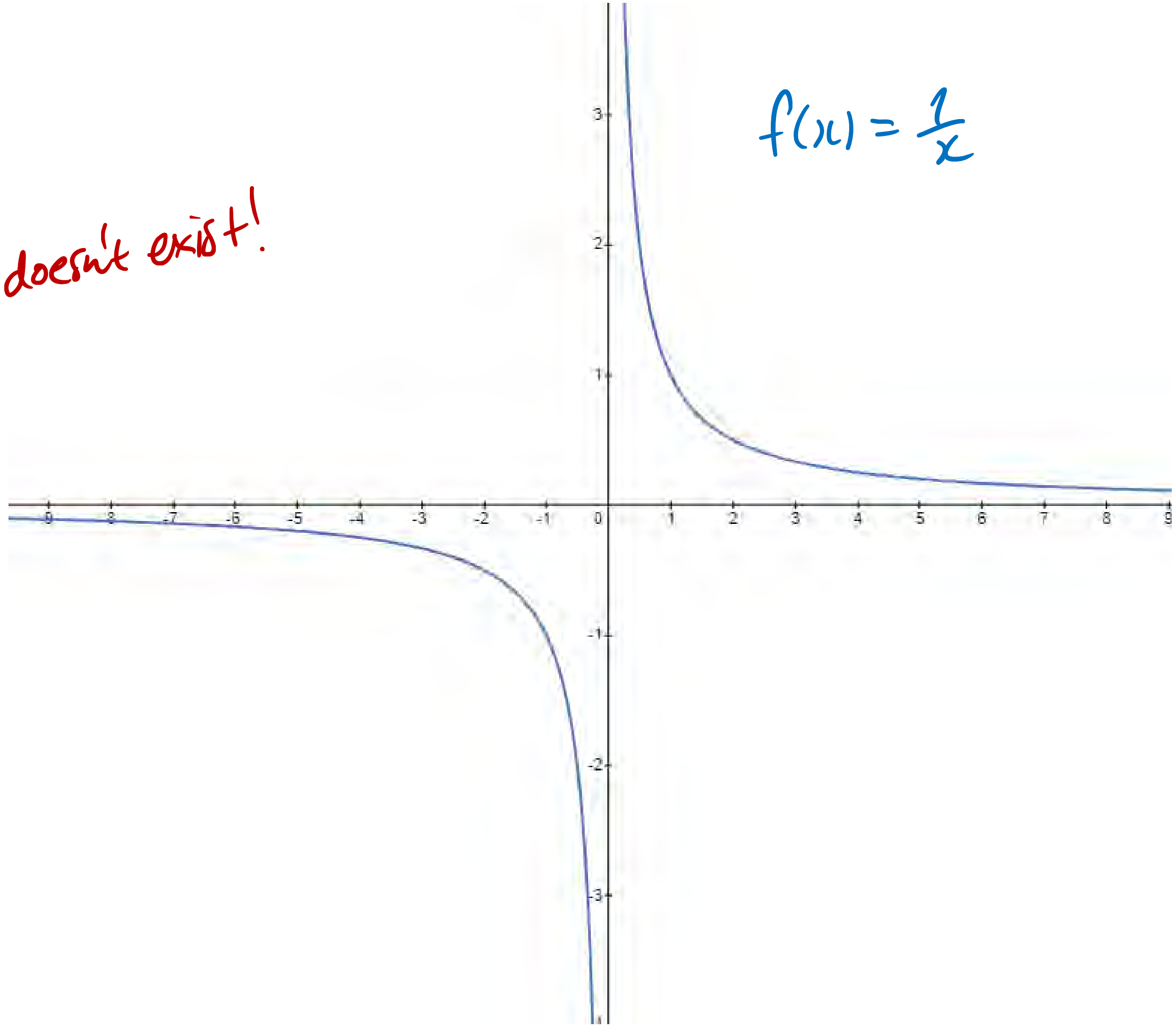
The definition of a derivative

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$f'(0) = \lim_{h \to 0} \frac{f(h) - f(0)}{h}$$

doesn't exist!

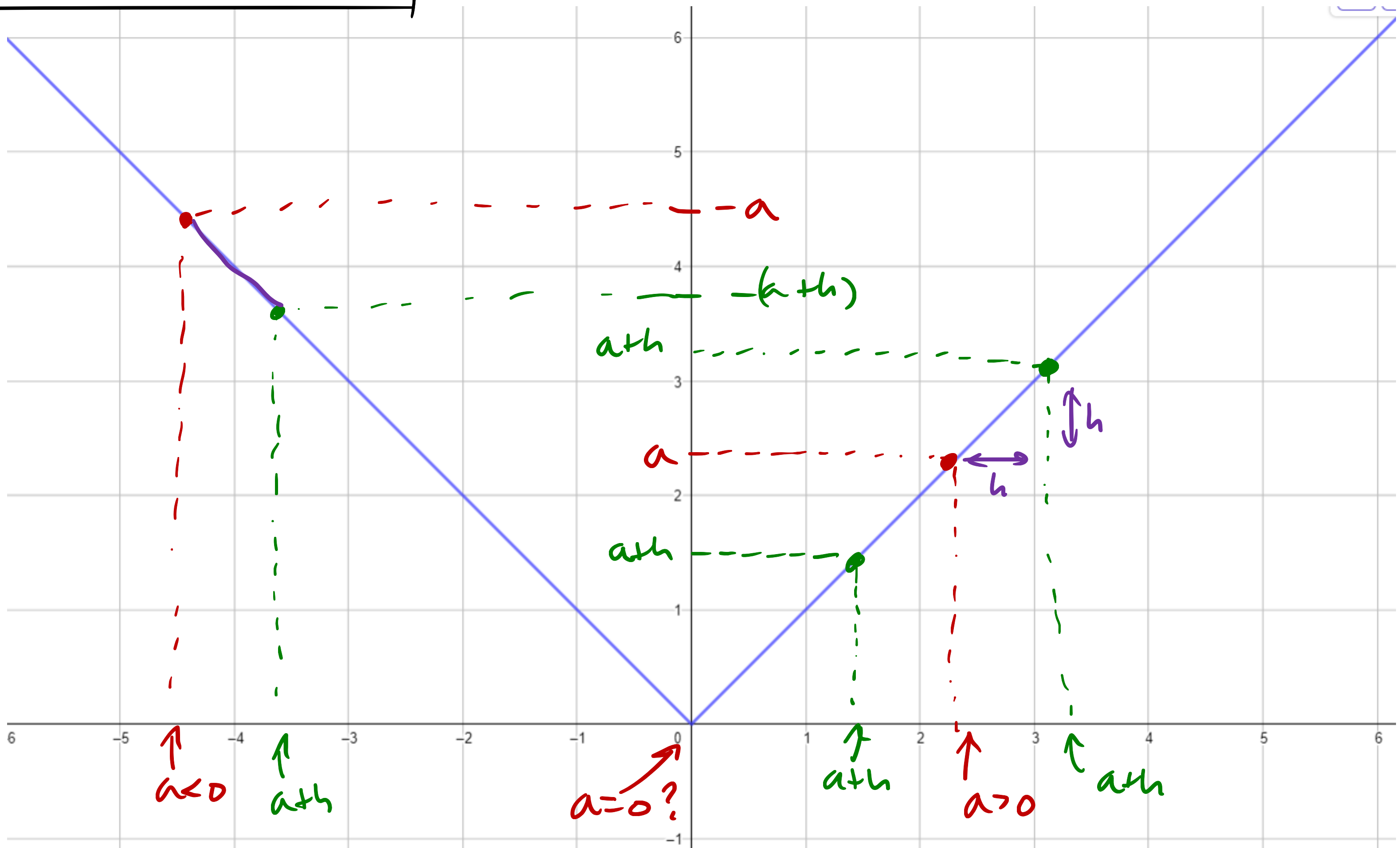
$f(x) = \frac{1}{x}$



The definition of a derivative

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$



if $a<0$, $f'(a) = -1$ if $a>0$, $f'(a) = 1$

The definition of a derivative

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

If $h > 0$,

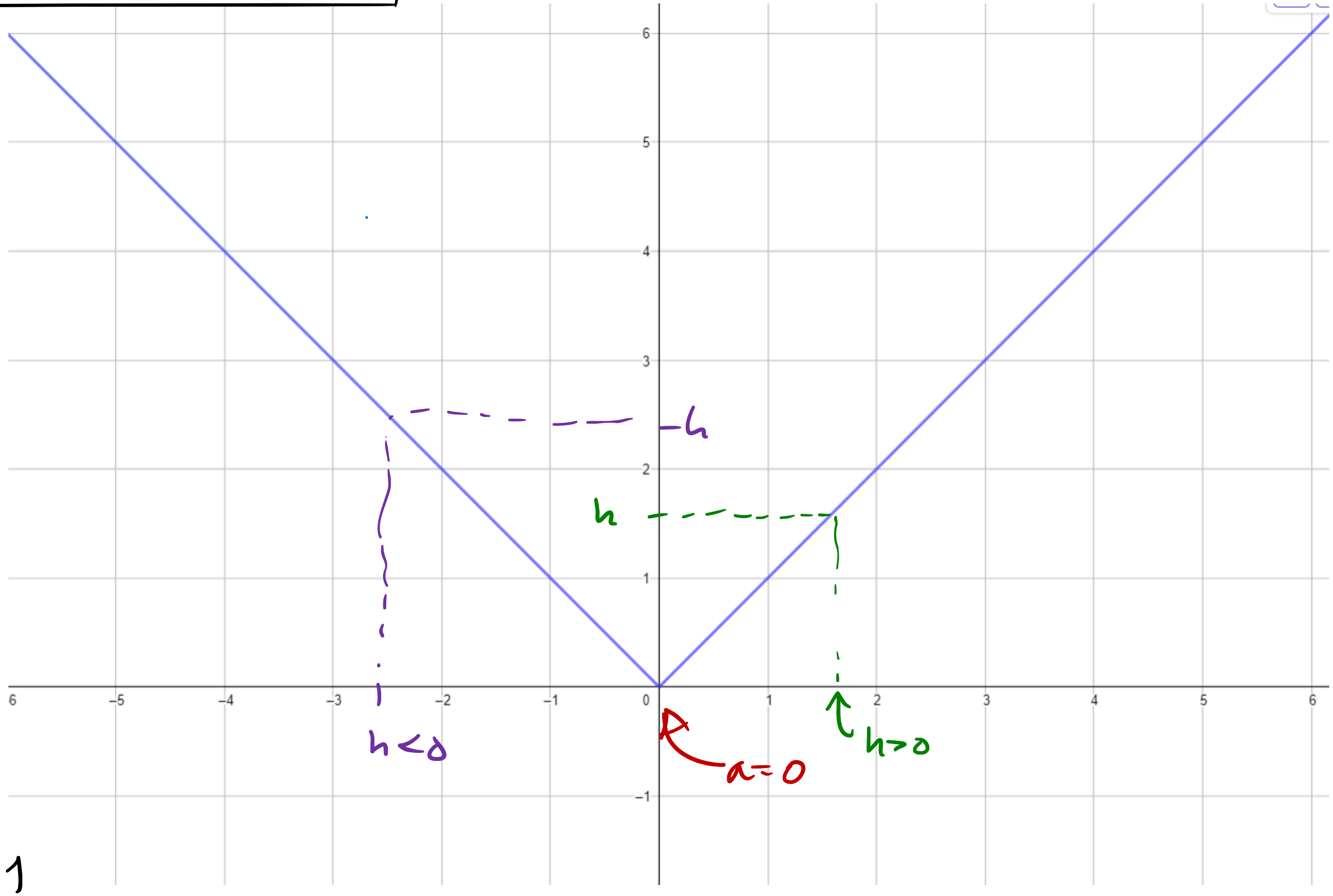
$$\frac{f(0+h) - f(0)}{h} = \frac{h - 0}{h} = 1$$

$$\lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h} = 1$$

If $h < 0$,

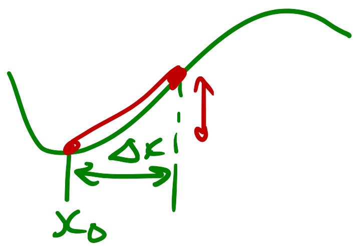
$$\frac{f(0+h) - f(0)}{h} = \frac{-h - 0}{h} = -1$$

$$\lim_{h \to 0^-} \frac{f(0+h) - f(0)}{h} = -1$$



Non-differentiability

$$f(x) = \mathrm{sign}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$$

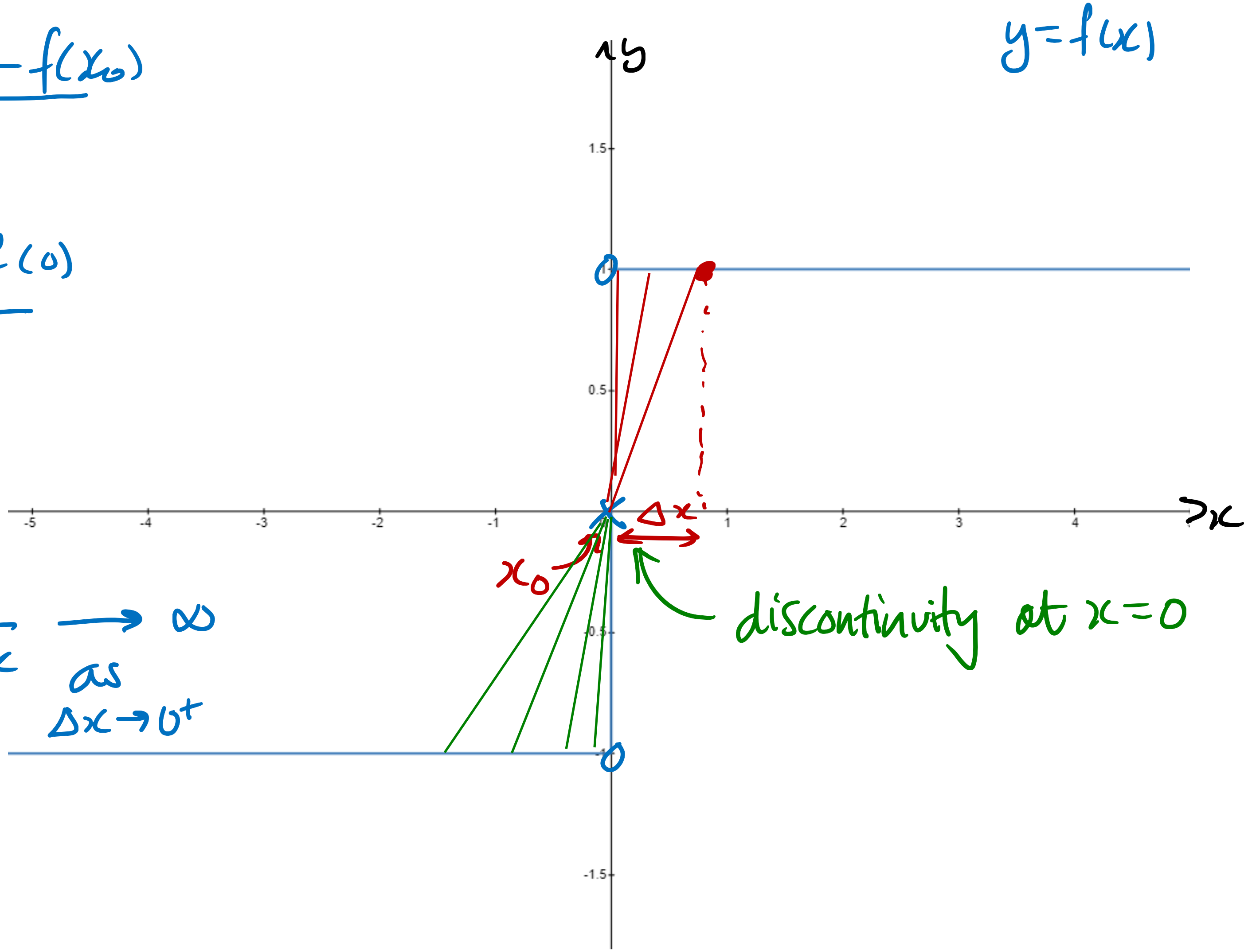


$$f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

$$f'(0) = \lim_{\Delta x \to 0} \frac{f(0+\Delta x) - f(0)}{\Delta x}$$

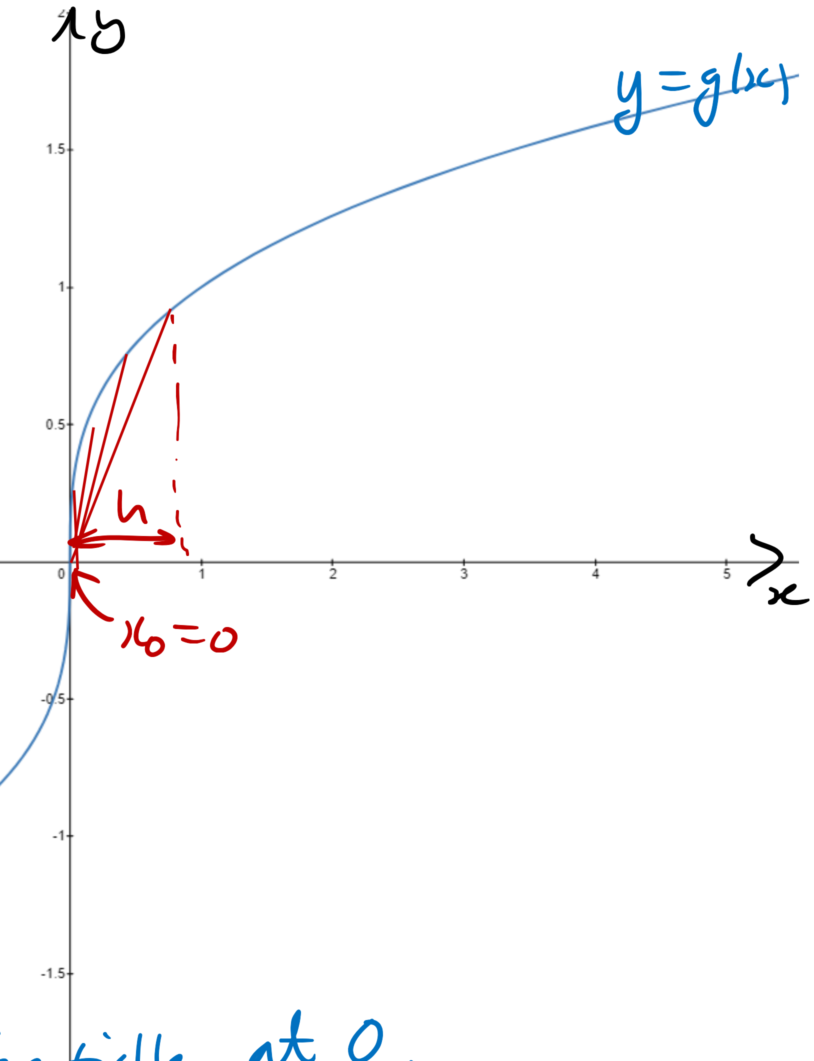
$$= \lim_{\Delta x \to 0} \frac{f(\Delta x)}{\Delta x}$$

if $\Delta x > 0$, then $\dfrac{f(\Delta x)}{\Delta x} = \dfrac{1}{\Delta x} \longrightarrow \infty$ as $\Delta x \to 0^+$



Non-differentiability

$g(x) = x^{1/3} = \sqrt[3]{x}$



$$\frac{g(0+h)-g(0)}{h} = \frac{h^{1/3} - 0}{h}$$

$$= \frac{h^{1/3}}{h}$$

$$= h^{-2/3}$$

$$= \frac{1}{h^{2/3}}$$

$$= \frac{1}{\sqrt[3]{(h^2)}}$$

$$\xrightarrow[\text{as } h \to 0]{} \infty$$

So $\lim_{h \to 0} \frac{g(0+h)-g(0)}{h}$ doesn't exist.

So g is not differentiable at 0.

Non-differentiability