The definition of a derivative

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

The definition of a derivative

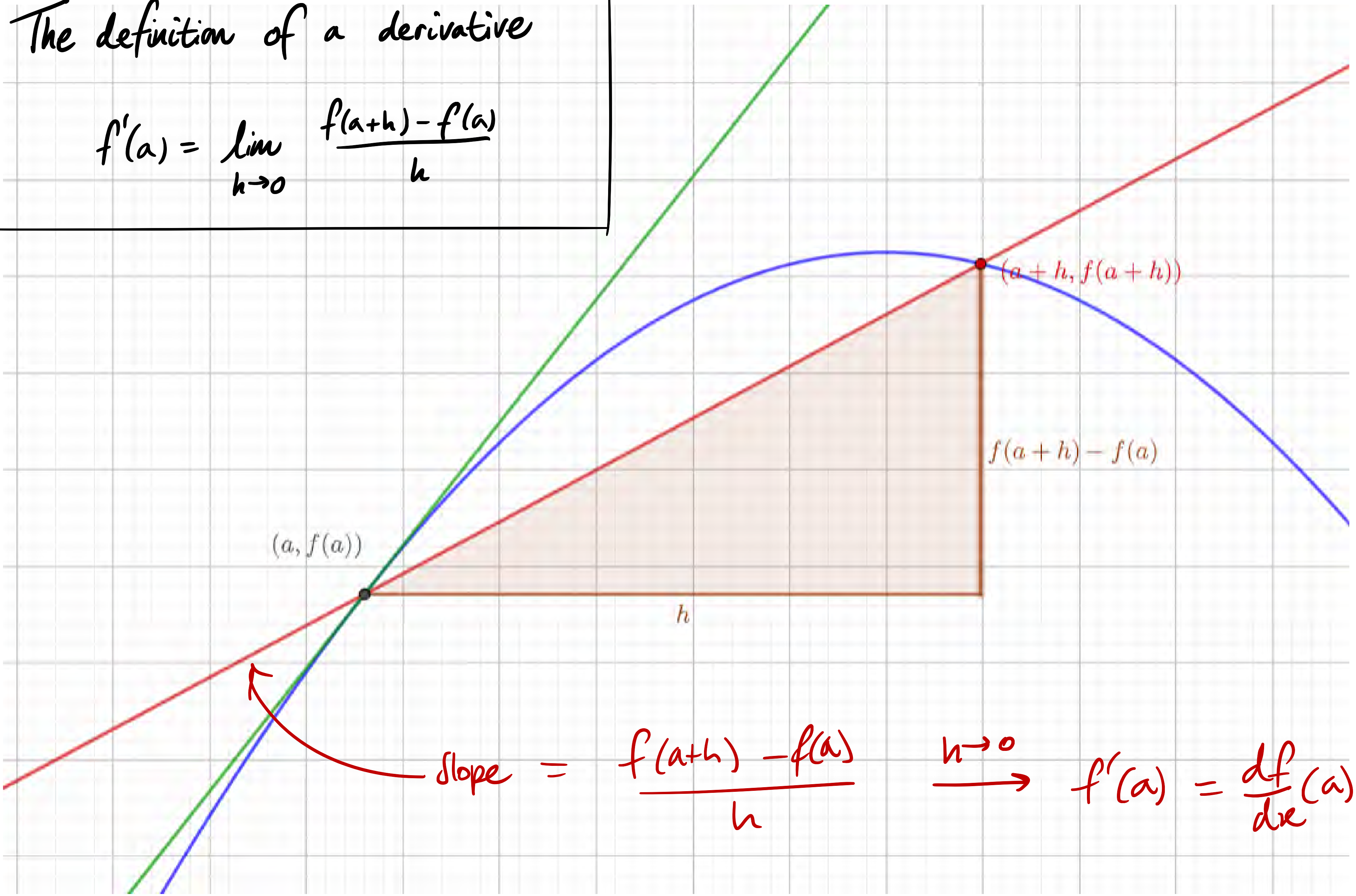
The derivative of function f at the point a is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \qquad \text{if the limit exists}$$

$$= \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

The definition of a derivative

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f'(a)}{h}$$

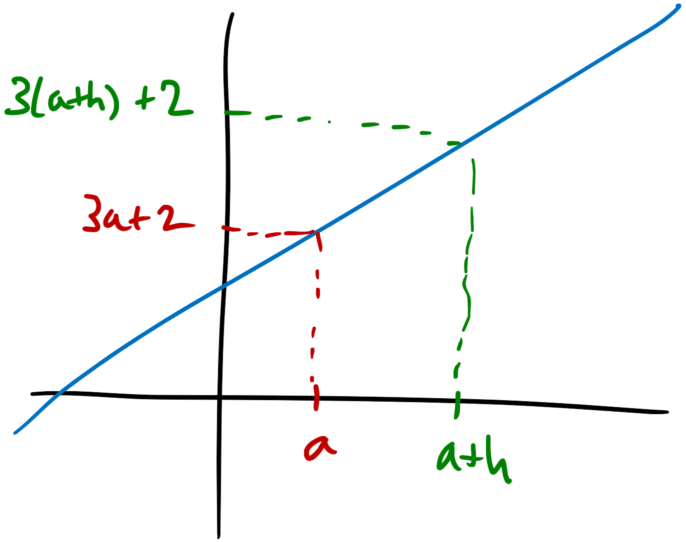


$$\text{slope} = \frac{f(a+h) - f(a)}{h} \xrightarrow{h \to 0} f'(a) = \frac{df}{dx}(a)$$

The definition of a derivative

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$f(x) = 3x + 2$



$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{(3a+3h+2) - (3a+2)}{h}$$

$$= \lim_{h \to 0} \frac{3h}{h} = \lim_{h \to 0} 3 = 3$$

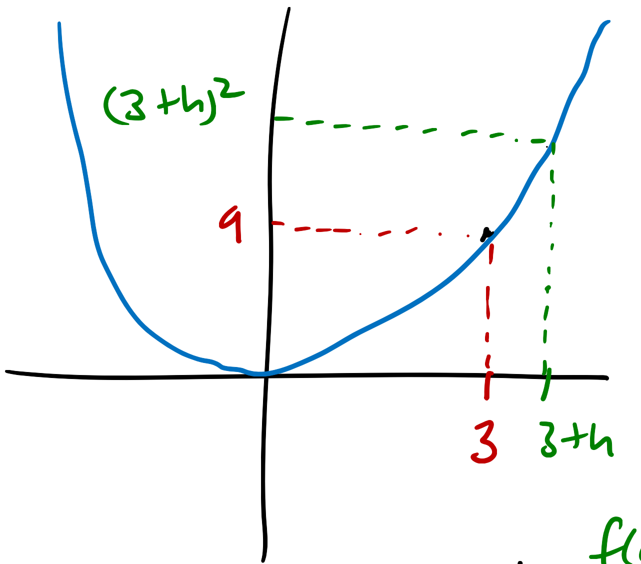
so $f'(a) = 3$ for any a

The definition of a derivative

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

$f(x) = x^2$

$f'(x) = 2x$



$$\lim_{h\to 0} \frac{f(3+h)-f(3)}{h} = \lim_{h\to 0} \frac{(9+6h+h^2)-9}{h}$$

$$= \lim_{h\to 0} \frac{6h+h^2}{h}$$

$$= \lim_{h\to 0} (6+h) = 6$$

So $f'(3) = 6$

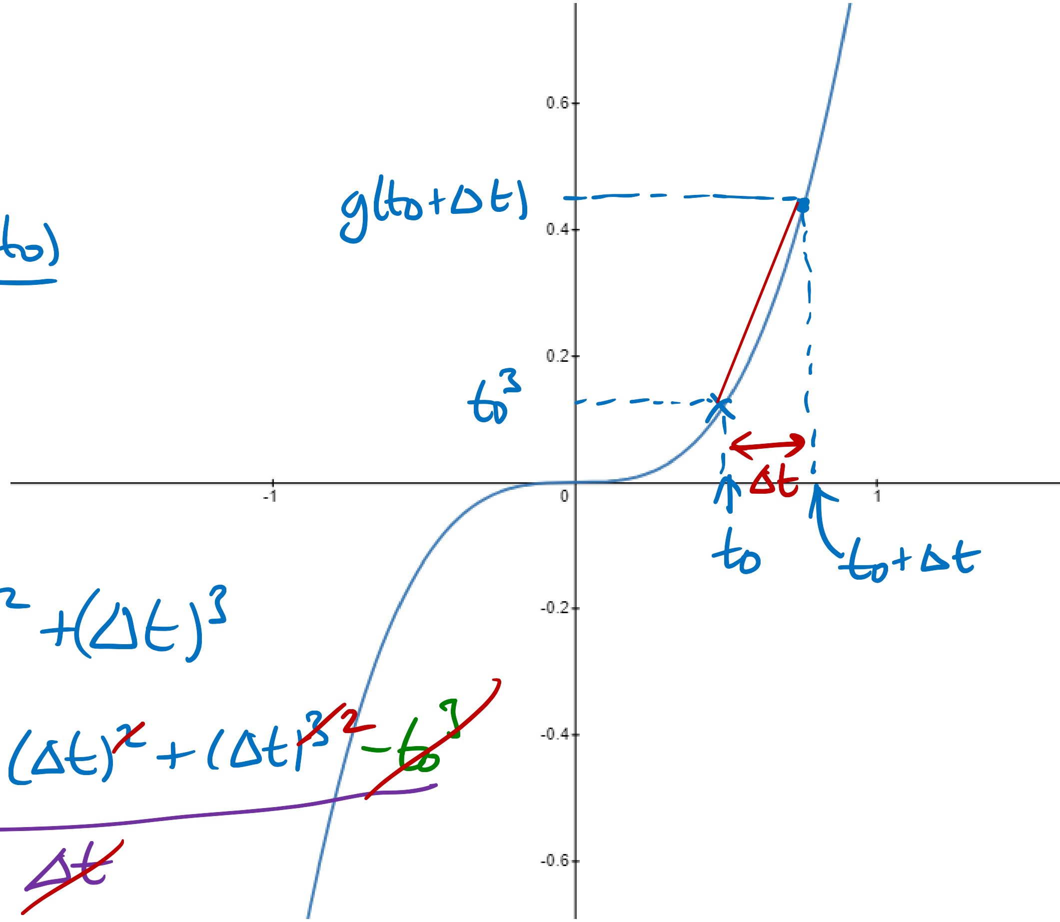
$$\lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{(a+h)^2-a^2}{h} = \lim_{h\to 0} \frac{a^2+2ah+h^2-a^2}{h} = \lim_{h\to 0}(2a+h) = 2a$$

$f'(a) = 2a$

The definition of a derivative

Find $\left.\frac{dg}{dt}\right|_{t=t_0}$ where $g(t) = t^3$.

$$g'(t_0) = \lim_{\Delta t \to 0} \frac{g(t_0+\Delta t) - g(t_0)}{\Delta t}$$



$$g(t_0+\Delta t) = (t_0+\Delta t)^3$$

$$= t_0^3 + 3t_0^2(\Delta t) + 3t_0(\Delta t)^2 + (\Delta t)^3$$

$$\frac{g(t_0+\Delta t) - g(t_0)}{\Delta t} = \frac{\cancel{t_0^3} + 3t_0^2\cancel{(\Delta t)} + 3t_0(\Delta t)^{\cancel{2}} + (\Delta t)^{\cancel{3}\,2} - \cancel{t_0^3}}{\cancel{\Delta t}}$$

$$= 3t_0^2 + 3t_0(\Delta t) + (\Delta t)^2$$

$$\xrightarrow[\text{as } \Delta t \to 0]{} 3t_0^2 + 3t_0 \times 0 + 0^2 = 3t_0^2$$

$$\left.\frac{dg}{dt}\right|_{t=t_0} = \underline{\underline{3t_0^2}}$$