

# Inverses

If  $y = 3x + 2$ , complete the table of values

$x$	$y$
0	
1	
$\frac{1}{2}$	$\frac{7}{2}$
	7
	-2
	3

Find the inverses of  $f(x) = x + 3$ ,  $g(t) = 5t$ ,  $\varphi(a) = a^3$ ,  $\psi(y) = 2^y$ ,  $h(z) = \frac{1}{z}$

Find  $h^{-1}$  where  $h(t) = \frac{5t^3 + 7}{2}$

If  $\pi = AK^{1/4}$  where  $A$  is constant, find  $K$  in terms of  $\pi$

# Inverses

If  $y = 3x + 2$ , complete the table of values

$x$	$y$
0	2
1	5
$\frac{1}{2}$	$\frac{7}{2}$
$\frac{5}{3}$	7
$-\frac{4}{3}$	-2
$\frac{1}{3}$	3

$y = 3 \times 0 + 2 = 0 + 2 = 2$

$y = 3 \times 1 + 2 = 3 + 2 = 5$

$x = \frac{(-2) - 2}{3} = -\frac{4}{3}$

$x = \frac{3 - 2}{3} = \frac{1}{3}$

$y = 3x + 2$   
 $\downarrow -2$   
 $y - 2 = 3x$   
 $\downarrow \div 3$   
 $\frac{y - 2}{3} = x$

$7 = 3x + 2$   
 $\downarrow -2$   
 $5 = 3x$   
 $\downarrow \div 3$   
 $\frac{5}{3} = x$

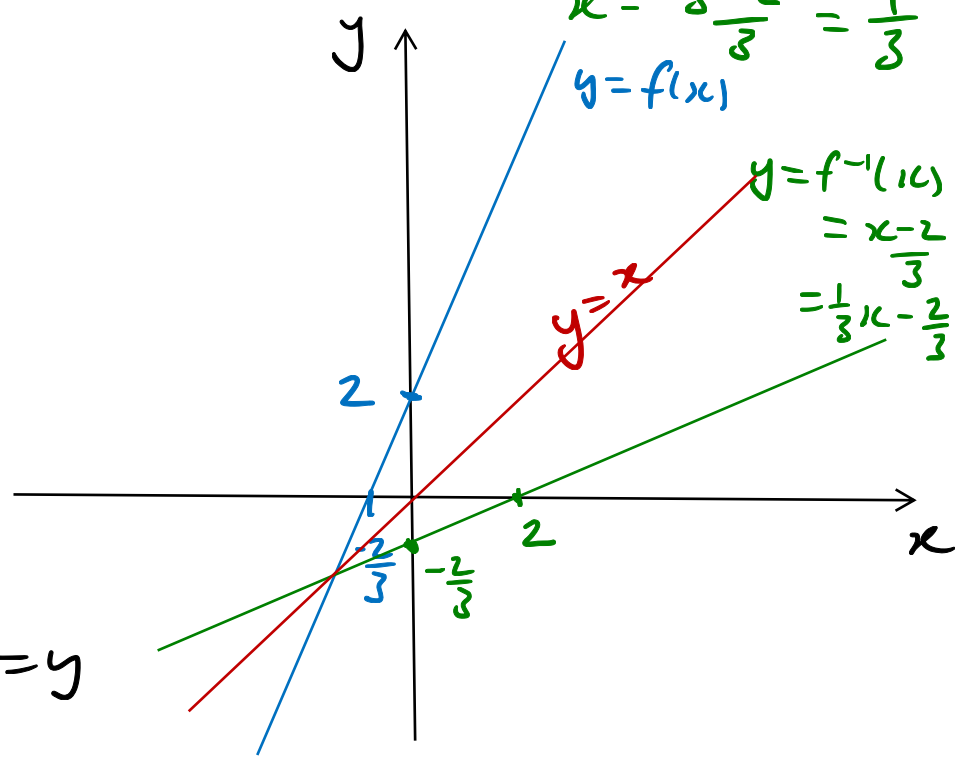
$x = \frac{y - 2}{3}$

$y = f(x) = 3x + 2$

$x = f^{-1}(y) = \frac{y - 2}{3}$   
↳ inverse of  $f$

$f(f^{-1}(y)) = 3 \times f^{-1}(y) + 2 = 3 \left( \frac{y - 2}{3} \right) + 2 = y$

$f^{-1}(f(x)) = \frac{f(x) - 2}{3} = \frac{(3x + 2) - 2}{3} = x$



# Inverses

Find the inverses of

$$f(x) = x + 3$$

$$\begin{aligned}y &= f(x) \\y &= x + 3 \\y - 3 &= x\end{aligned}$$

$$f^{-1}(y) \neq (f(y))^{-1} = \frac{1}{f(y)}$$

$$x = \underline{f^{-1}(y) = y - 3}$$

$$g(t) = 5t$$

$$\begin{aligned}u &= g(t) \\u &= 5t \\ \frac{u}{5} &= t\end{aligned}$$

$$t = \underline{g^{-1}(u) = \frac{u}{5}}$$

$$\varphi(a) = a^3$$

$$\begin{aligned}b &= \varphi(a) \\b &= a^3 \\ \sqrt[3]{b} &= a\end{aligned}$$

$$a = \varphi^{-1}(b) = \sqrt[3]{b}$$

$$\psi(y) = 2^y$$

$$\begin{aligned}z &= \psi(y) \\z &= 2^y\end{aligned}$$

$$y = \psi^{-1}(z) = \log_2(z)$$

$$h(z) = \frac{1}{z}$$

$$\begin{aligned}\log_2 z &= y \\x &= h(z) \\x &= \frac{1}{z} \\zx &= 1 \\z &= \frac{1}{x}\end{aligned}$$

$$z = h^{-1}(x) = \frac{1}{x} = h(x)$$

# Inverses

Find  $h^{-1}$  where  $h(t) = \frac{5t^3 + 7}{2}$

$$u = h(t)$$

Aim  $t = h^{-1}(u)$

$$u = \frac{5t^3 + 7}{2}$$

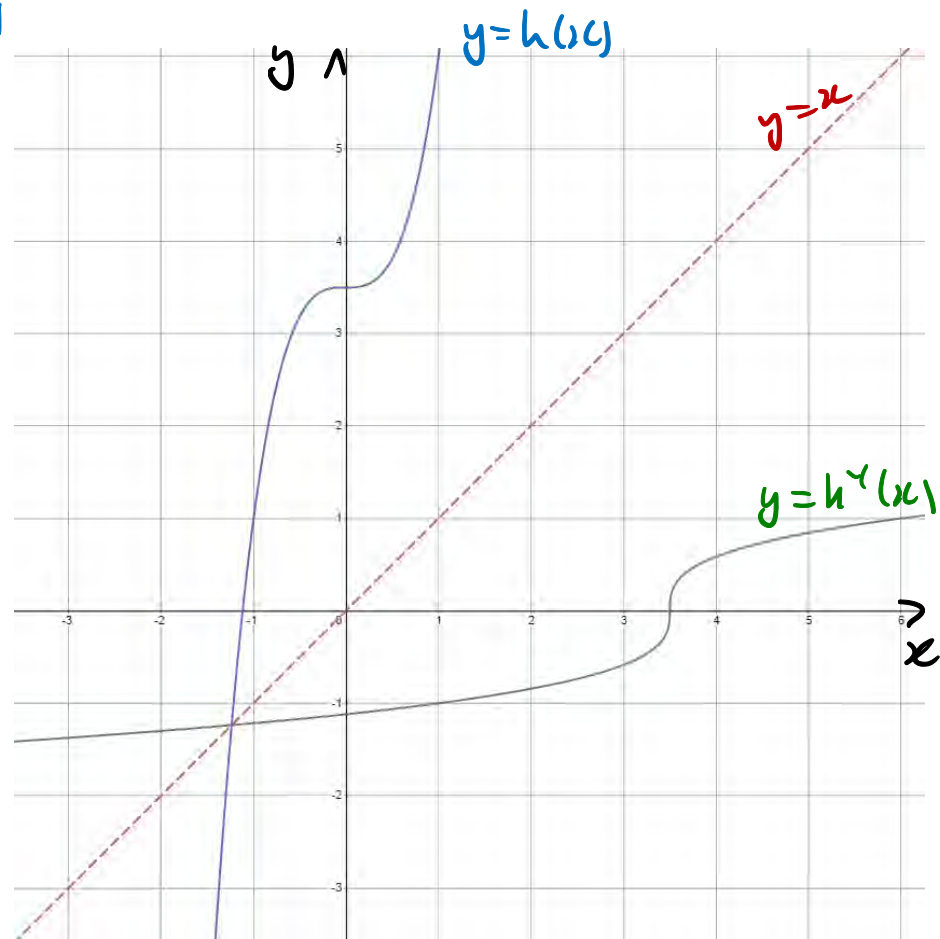
$$h^{-1}(u) = \sqrt[3]{\frac{2u - 7}{5}}$$

$$2u = 5t^3 + 7$$

$$2u - 7 = 5t^3$$

$$\frac{2u - 7}{5} = t^3$$

$$\sqrt[3]{\frac{2u - 7}{5}} = t$$



# Inverses

If  $\pi = AK^{1/4}$  where  $A$  is constant, find  $K$  in terms of  $\pi$

$$\pi = f(K) = AK^{1/4}, \quad \underline{\text{AIM}} \quad K = f^{-1}(\pi)$$

$$\pi = AK^{1/4}$$

$$\frac{\pi}{A} = K^{1/4} = \sqrt[4]{K}$$

$$\left(\frac{\pi}{A}\right)^4 = K$$

$$K = \frac{\pi^4}{A^4}$$

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# Inverses

Is  $f(x) = x^2$  invertible?

$$y = f(x) \quad \text{Ans} \quad x = f^{-1}(y)$$

$$y = x^2$$

$$\sqrt{y} = x$$

$$(-2)^2 = 4$$

$$f^{-1}(f(x)) = x$$

$$\underline{x=2} \quad f^{-1}(f(2)) = 2$$

$$f^{-1}(4) = 2$$

$$\underline{x=-2} \quad f^{-1}(f(-2)) = -2$$

$$f^{-1}(4) = -2$$

