Completing the square

$$ax^2+bx+c = a(x-x_0)^2 + y_0$$

Solve $x^2 + 6x + 5 = 0$

Solve $-5t^2 + 15t + 7 = 3$

Find the minimum value of $y = 3x^2 - 12x - 7$

Suppose that $\pi = -5p^2 + 30p - 21$. For which values p is $\pi > 4$?

Completing the square

$$ax^2+bx+c = a(x-x_0)^2 + y_0$$

Solve $1x^2+6x+5=0$ (1: monic)

$$(x+3)^2-9+5=0$$

$$(x+3)^2-4=0$$

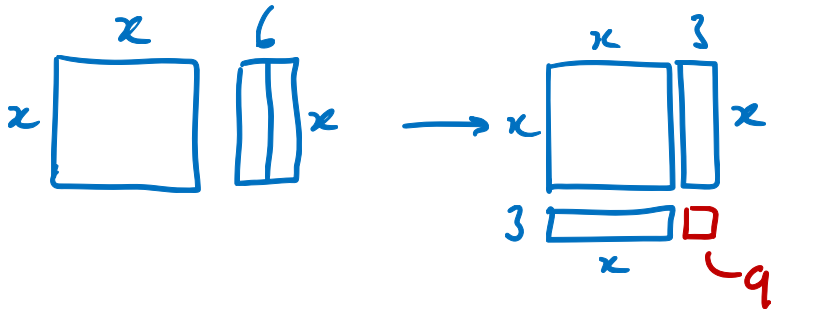
$$(x+3)^2=4$$

$$x+3=\pm 2$$

$x+3=2 \Rightarrow x=-1$

OR

$x+3=-2 \Rightarrow x=-5$



$x^2+6x = (x+3)^2-9$

x^2+6x+9

x^2+bx+c

$$\left(x+\frac{b}{2}\right)^2 = x^2+bx+\left(\frac{b}{2}\right)^2$$

$$x^2+bx+c = \left(x+\frac{b}{2}\right)^2-\left(\frac{b}{2}\right)^2+c$$

Completing the square

$$ax^2+bx+c = a(x-x_0)^2 + y_0$$

Solve $-5t^2+15t+7=3$

$$-5t^2+15t=-4$$

$$t^2-3t=\frac{4}{5}$$

$$(t-\tfrac{3}{2})^2 = t^2-3t+\frac{9}{4}$$

$$(t-3/2)^2-\frac{9}{4}=\frac{4}{5}$$

$$(t-3/2)^2=\frac{4}{5}+\frac{9}{4}$$

$$(t-3/2)^2=\frac{16}{20}+\frac{45}{20}=\frac{61}{20}$$

$$t-\tfrac{3}{2}=\pm\sqrt{\tfrac{61}{20}}$$

$$t=\tfrac{3}{2}\pm\sqrt{\tfrac{61}{20}}$$

$$-5t^2+15t+7=3$$

$$-5(t^2-3t)+7=3$$

$$-5\left((t-\tfrac{3}{2})^2-\tfrac{9}{4}\right)+7=3$$

$$-5\left[(t-3/2)^2-\tfrac{9}{4}\right]=-4$$

$$(t-\tfrac{3}{2})^2-\tfrac{9}{4}=\tfrac{4}{5}$$

Completing the square

$$ax^2+bx+c = a(x-x_0)^2 + y_0$$

Find the minimum value of $y = 3x^2 - 12x - 7$

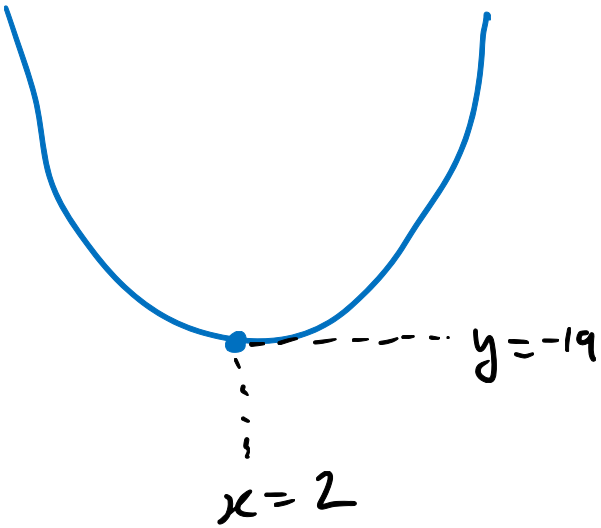
$$y = 3(x^2-4x) - 7$$

$$(x-2)^2 = x^2-4x+4$$

$$y = 3((x-2)^2-4) - 7$$

$$= 3(x-2)^2 - 12 - 7$$

$$= 3(x-2)^2 - 19$$

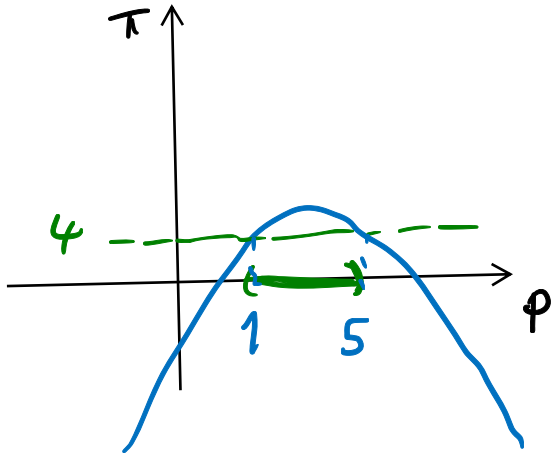


if $x=2$, then $y = 3\times0^2-19 = -19$

if $x\neq2$, then $y > 3\times0^2-19$

Completing the square

$$ax^2+bx+c = a(x-x_0)^2 + y_0$$



Suppose that $\pi = -5p^2+30p-21$. For which values p is $\pi > 4$?

$$\pi = -5(p^2-6p) - 21$$

$$= -5((p-3)^2-9) - 21$$

$$= -5(p-3)^2+45-21$$

$$= -5(p-3)^2+24$$

$(p-3)^2 = p^2-6p+9$

$\pi > 4 \iff -5(p-3)^2+24 > 4$ ↘ -24

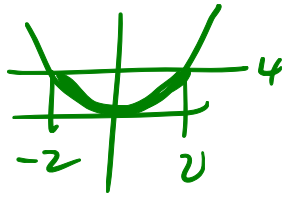
$\iff -5(p-3)^2 > -20$ ↘ $\div -5$

$\iff (p-3)^2 < 4$

$\iff -2 < p-3 < 2$ ↘ $+3$

$\iff 1 < p < 5$

$t^2 < 4$



$\Updownarrow$

$-2 < t < 2$