The quadratic formula

$$ax^2 + bx + c = 0 \iff x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve $6x^2 - x - 2 = 0$

Solve $12 - 6t - 9t^2 = -9t + 2$

Find the zeros of $25z^2 - 10z + 1$

Solve $6x^2 + 5x + 10 = 2x^2 - 7x - 3$

For how many values p do we get $3p^2 + 5p - 2 = 4$

For which values of q is $\pi = -2q^2 + 5q + 3$ negative
& what is the maximum value of π?

The quadratic formula

discriminant

$\Delta = b^2 - 4ac$

$$ax^2 + bx + c = 0 \iff x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

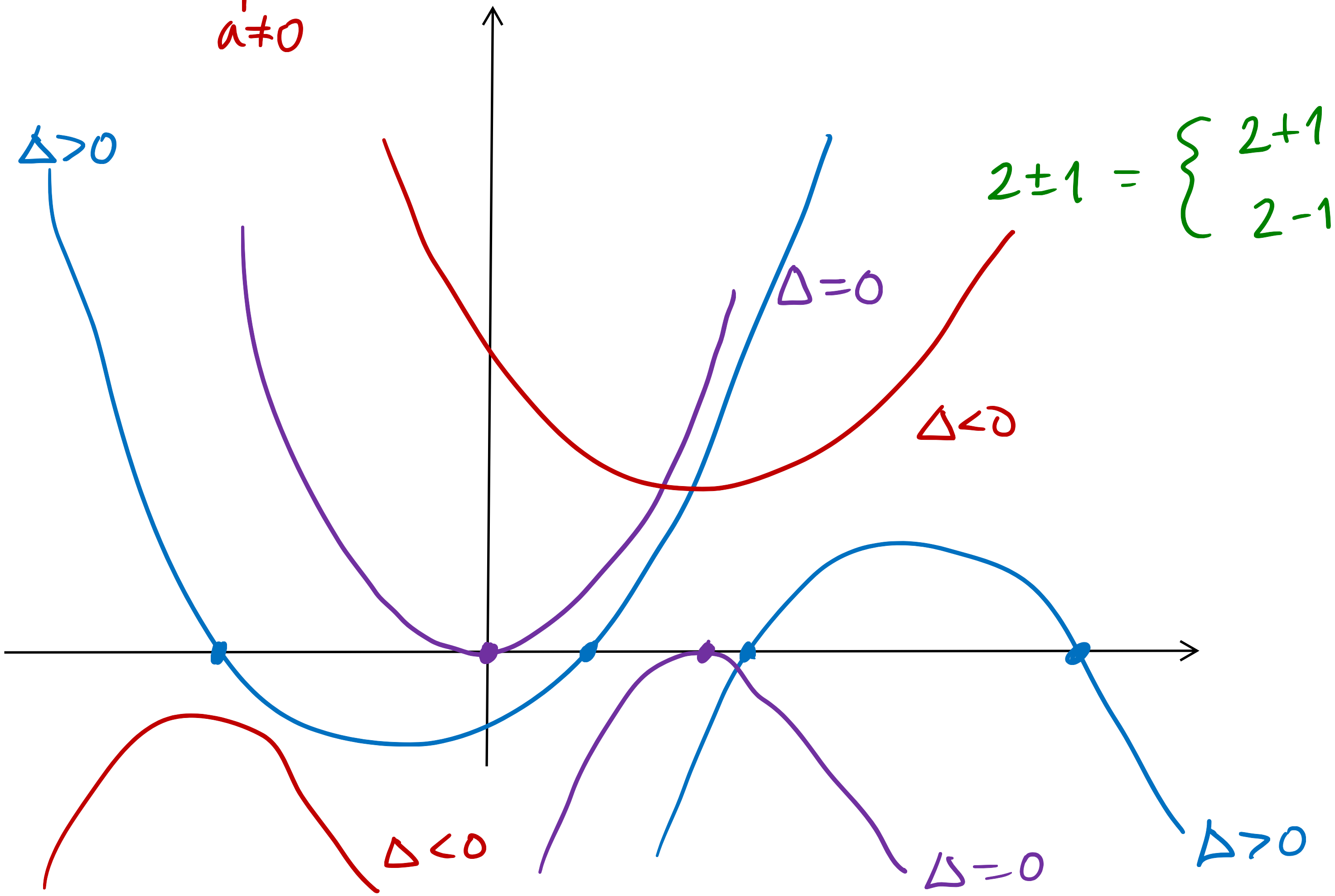
$a \neq 0$

$\Delta > 0 \Rightarrow 2$ roots

$\Delta = 0 \Rightarrow 1$ root

$\Delta < 0 \Rightarrow 0$ real solutions

$$2 \pm 1 = \begin{cases} 2+1 = 3 \\ 2-1 = 1 \end{cases}$$



The quadratic formula

$$ax^2 + bx + c = 0 \iff x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve $\underline{6}x^2 \underline{-} x \underline{-2} = 0$

$a = 6$

$b = -1$

$c = -2$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 6 \times (-2)}}{2 \times 6}$$

$$= \frac{1 \pm \sqrt{1 + 48}}{12}$$

$$= \frac{1 \pm \sqrt{49}}{12} = \frac{1 \pm 7}{12}$$

2 solutions: ① $x = \frac{1+7}{12} = \frac{8}{12} = \frac{2}{3}$

② $x = \frac{1-7}{12} = \frac{-6}{12} = \frac{-1}{2}$

double check

$$6x^2 - x - 2 = (3x-2)(2x+1)$$

$$6x^2 - x - 2 = 0$$

$$\iff (3x-2)(2x+1) = 0$$

$$\iff \begin{cases} 3x - 2 = 0 \iff 3x = 2 \iff x = 2/3 \\ \text{OR} \\ 2x + 1 = 0 \iff 2x = -1 \iff x = -1/2 \end{cases}$$

The quadratic formula

$$ax^2 + bx + c = 0 \iff x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve $\quad 12 - 6t - 9t^2 = -9t + 2$

$\quad\quad -2 \quad +9t \quad\quad\quad\quad +9t \quad -2$

$$10 + 3t - 9t^2 = 0$$

$$\underbrace{-9}_{a}t^2 + \underbrace{3}_{b}t + \underbrace{10}_{c} = 0$$

$$\begin{aligned} \Delta = b^2 - 4ac &= 3^2 - 4 \times (-9) \times 10 \\ &= 9 + 360 \\ &= 369 = 9 \times 41 \end{aligned}$$

$$\begin{aligned} 369 &= 360 + 9 \\ &= 40 \times 9 + 1 \times 9 = 41 \times 9 \end{aligned}$$

$$\sqrt{9 \times 41} = \sqrt{9}\sqrt{41} = 3\sqrt{41}$$

$$t = \frac{-3 \pm \sqrt{9 \times 41}}{-18}$$

$$t = \frac{-3 \pm 3\sqrt{41}}{-18}$$

$$t = \frac{-1 \pm \sqrt{41}}{-6} = \frac{1 \mp \sqrt{41}}{6}$$

2 solutions:

① $t = \dfrac{-1 + \sqrt{41}}{-6} = \dfrac{1 - \sqrt{41}}{6}$

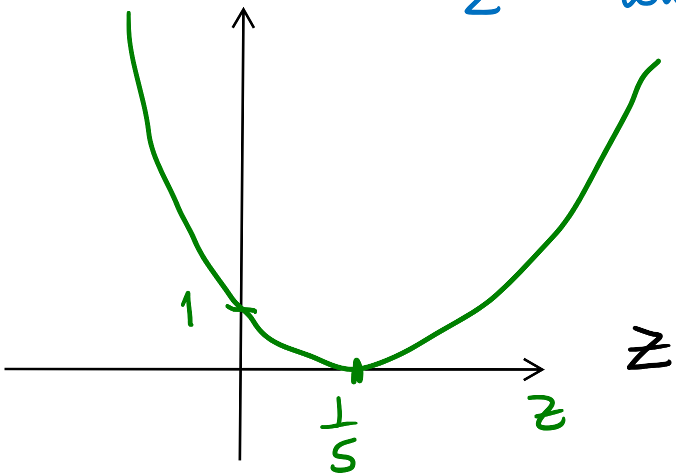
② $t = \dfrac{-1 - \sqrt{41}}{-6} = \dfrac{1 + \sqrt{41}}{6}$

The quadratic formula

$$ax^2 + bx + c = 0 \iff x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Find the zeros of $25z^2 - 10z + 1$

z where $\underbrace{25z^2}_{a=25} \underbrace{- 10z}_{b=-10} \underbrace{+1}_{c=1} = 0$



$$z = \frac{-(-10) \pm \sqrt{(-10)^2 - 4 \times 25 \times 1}}{2 \times 25}$$

$$= \frac{10 \pm \sqrt{100 - 100}}{50}$$

$$= \frac{10 \pm \sqrt{0}}{50} = \frac{10 \pm 0}{50} = \frac{10}{50} = \frac{1}{5}$$

repeated root

check

$$(5z-1)^2 = (5z-1)(5z-1)$$
$$= 25z^2 - 10z + 1$$

$$(5z-1)^2 = 0 \iff 5z - 1 = 0$$
$$\iff 5z = 1 \iff z = 1/5$$

The quadratic formula

$$ax^2 + bx + c = 0 \iff x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve $\quad 6x^2 + 5x + 10 = 2x^2 - 7x - 3$

$\quad -2x^2 \quad +7x + 3 \qquad -2x^2 + 7x + 3$

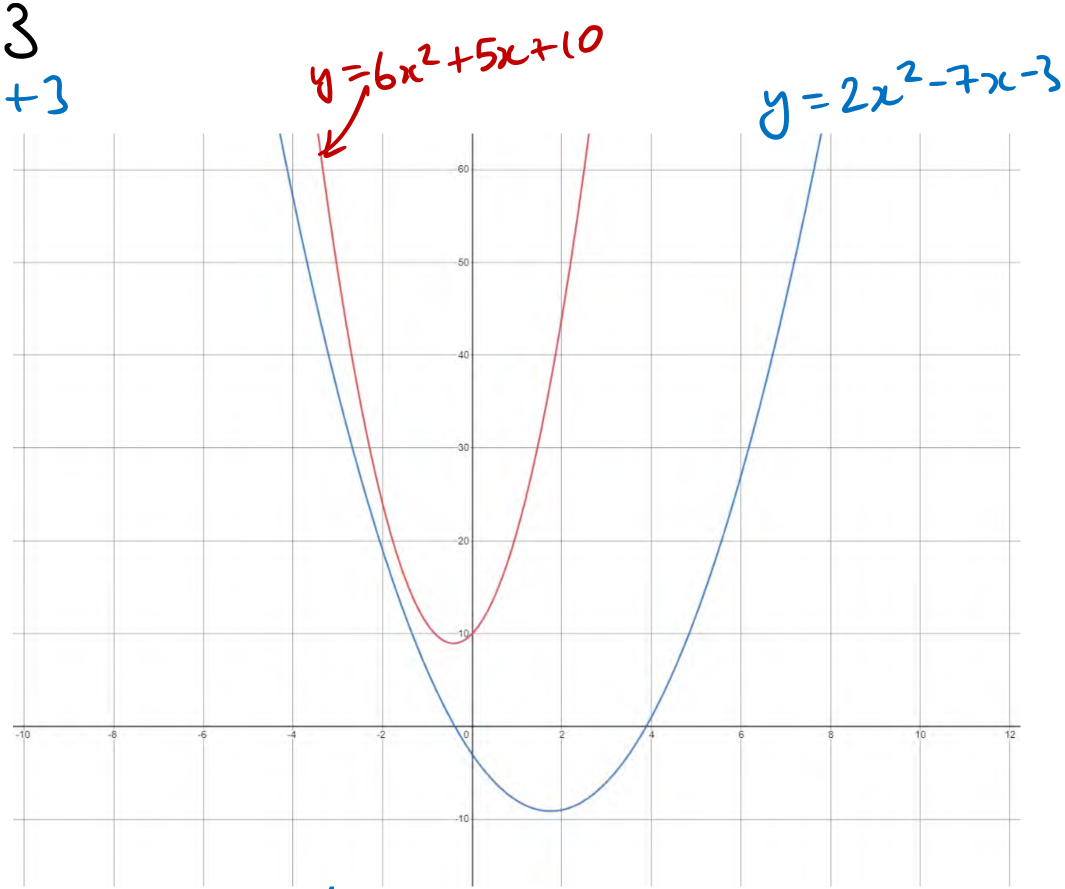
$$\underbrace{4}_{a=4}x^2 + \underbrace{12}_{b=12}x + \underbrace{13}_{c=13} = 0$$

$$x = \frac{-12 \pm \sqrt{12^2 - 4\times 4\times 13}}{2\times 4}$$

$$= \frac{-12 \pm \sqrt{144 - 208}}{8}$$

$$= \frac{-12 \pm \sqrt{-64}}{8}$$

no real solutions!



$4\times 4\times 13$
$= 16\times 13 = 208$

	10	6
10	100	60
3	30	18

The quadratic formula

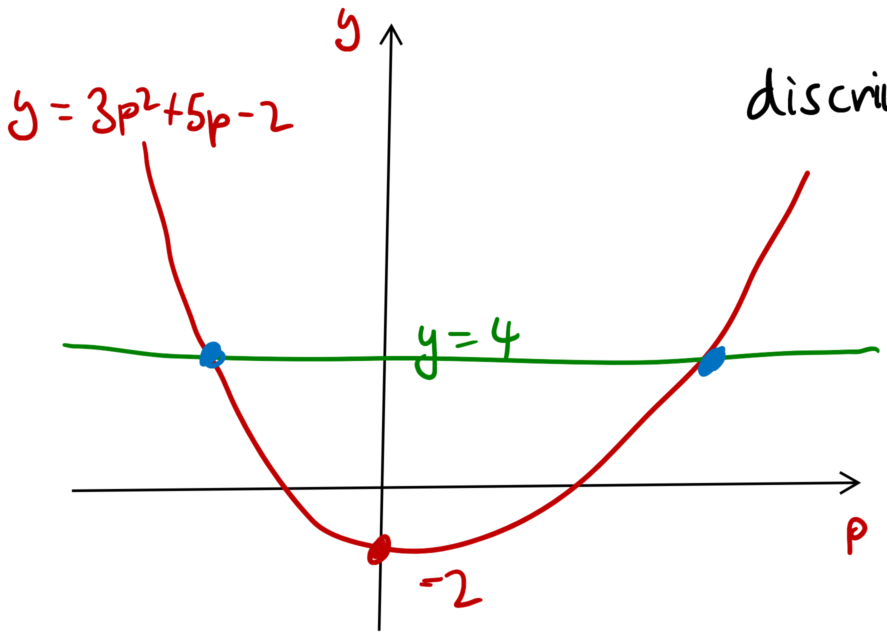
$$ax^2+bx+c=0 \iff x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

For how many values p do we get $3p^2+5p-2=4$

$$-4 \qquad -4$$

$$\underline{3}p^2+\underline{5}p\underline{-6}=0$$

$a=3 \qquad b=5 \qquad c=-6$



discriminant

$$\Delta=b^2-4ac$$
$$=5^2-4\times3\times(-6)$$
$$=25+72$$
$$=97>0$$

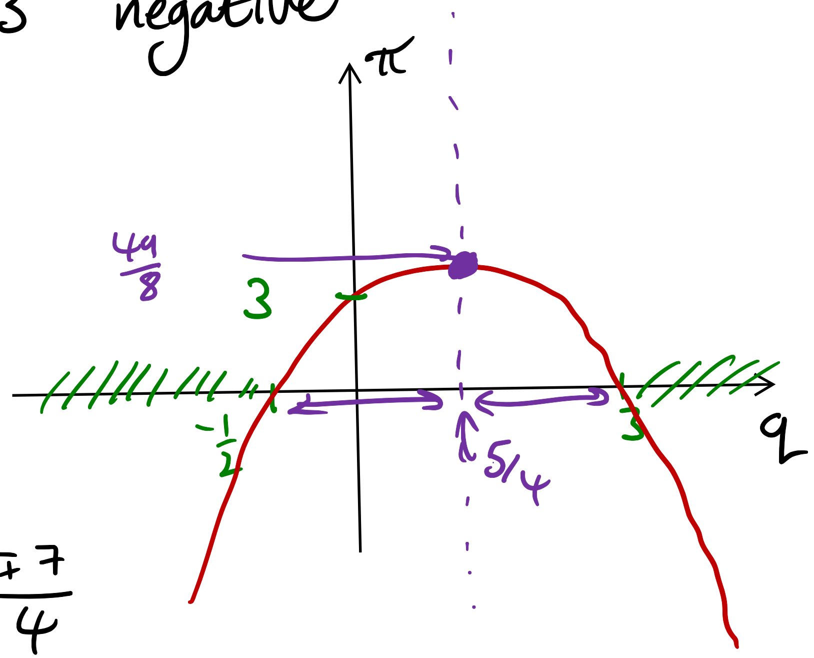
So 2 solutions

ie. 2 p values.

The quadratic formula

$$ax^2+bx+c=0 \iff x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

For which values of q is $\pi=-2q^2+5q+3$ negative & what is the maximum value of π?



$$\pi=0 \iff \underbrace{-2}_{a}q^2+\underbrace{5}_{b}q+\underbrace{3}_{c}=0$$

$$\iff q=\frac{-5\pm\sqrt{25+24}}{-4}$$

$$\iff q=\frac{-5\pm\sqrt{49}}{-4}=\frac{-5\pm7}{-4}=\frac{5\mp7}{4}$$

$$\iff q=\frac{5-7}{4}=\frac{-2}{4}=\frac{-1}{2} \quad \text{OR} \quad q=\frac{5+7}{4}=\frac{12}{4}=3$$

So $\pi<0$ if either $q<-1/2$ OR $q>3$

$$\frac{3+(-\frac{1}{2})}{2}=\frac{(5/2)}{2}=5/4$$

max at $q=5/4$, $\pi=-2(5/4)^2+5(5/4)+3$

$$=-\frac{50}{16}+\frac{25}{4}+3=\frac{-50}{16}+\frac{100}{16}+\frac{48}{16}=\frac{98}{16}=\frac{49}{8}$$