

Solving quadratics by factorization

Solve $x^2 - 2x - 15 = 0$

Find the roots of $2t^2 + 18t - 44 = 0$

For which z do we have $(z+2)(z-3) = -4$?

Solve $5x^2 - 9x - 10 = 3x^2 + 5x + 26$

Find all p for which $3p^2 - 2 = 5p$

Solve $x^2 - 6x + 2 = 0$

Solving quadratics by factorization

$$ab = 0 \quad \Rightarrow \quad \text{either } a=0 \quad \text{or } b=0 \quad (\text{or both})$$

$$(x-2)(x-7) = 0 \quad \Rightarrow \quad \begin{array}{l} \text{either } x-2=0 \quad \Leftrightarrow \quad x=2 \\ \text{or } x-7=0 \quad \Leftrightarrow \quad x=7 \end{array}$$

Solving quadratics by factorization

Solve $x^2 - 2x - 15 = 0$

AIM

$$\begin{aligned}x^2 - \underline{2x} - \underline{15} &= (x + a)(x + b) \\ &= x^2 + ax + bx + ab \\ &= x^2 + (\underline{a+b})x + \underline{ab}\end{aligned}$$

Need a, b where $a+b = -2$
 $ab = -15$

$$x^2 - 2x - 15 = (x-5)(x+3)$$

$$x^2 - 2x - 15 = 0 \Leftrightarrow (x-5)(x+3) = 0$$

$$\Leftrightarrow \begin{array}{l} x-5=0 \quad \text{ie } \underline{x=5} \\ \text{OR} \\ x+3=0 \quad \text{ie } \underline{x=-3} \end{array}$$

check

$x=5$:

$$\begin{aligned}x^2 - 2x - 15 \\ = 25 - 10 - 15 = 0 \checkmark\end{aligned}$$

$x=-3$

$$\begin{aligned}x^2 - 2x - 15 \\ = 9 + 6 - 15 = 0 \checkmark\end{aligned}$$

$$\begin{aligned}15 &= 1 \times 15 \\ &3 \times 5\end{aligned}$$

$$\begin{aligned}5 + (-3) &= 2 \times \\ (-5) + 3 &= -2 \checkmark\end{aligned}$$

Solving quadratics by factorization

Find the roots of $2t^2 + 18t - 44 = 0$

$$t^2 + \underline{9t} - \underline{22} = 0$$

$$(t + a)(t + b) = 0$$

$$(t - 2)(t + 11) = 0$$

$$t - 2 = 0 \quad \text{ie } \underline{t = 2}$$

or

$$t + 11 = 0 \quad \text{ie } \underline{t = -11}$$

$$\begin{aligned} a + b &= 9 \\ ab &= -22 \end{aligned}$$

$$1 \times 22$$

$$2 \times 11$$

$$2 + (-11) = -9 \quad \times$$

$$-2 + 11 = 9 \quad \checkmark$$

Solving quadratics by factorization

For which z do we have $(z+2)(z-3) = -4$?

$$z^2 - z - 6 = -4$$

$$z^2 - \underline{z} - \underline{2} = 0$$

$$(z - 2)(z + 1) = 0$$

$$\begin{aligned} a+b &= -1 \\ ab &= -2 \end{aligned}$$

$$z - 2 = 0 \quad \text{ie } \underline{z = 2}$$

or

$$z + 1 = 0 \quad \text{ie } \underline{z = -1}$$

Check

$$z = 2 \quad : \quad (z+2)(z-3) = 4 \times (-1) = -4 \quad \checkmark$$

$$z = -1 \quad : \quad (z+2)(z-3) = 1 \times (-4) = -4 \quad \checkmark$$

Solving quadratics by factorization

Solve

$$\begin{array}{r} 5x^2 - 9x - 10 = 3x^2 + 5x + 26 \\ \hline -3x^2 - 5x - 26 \quad -3x^2 - 5x - 26 \end{array}$$

$$2x^2 - 14x - 36 = 0$$

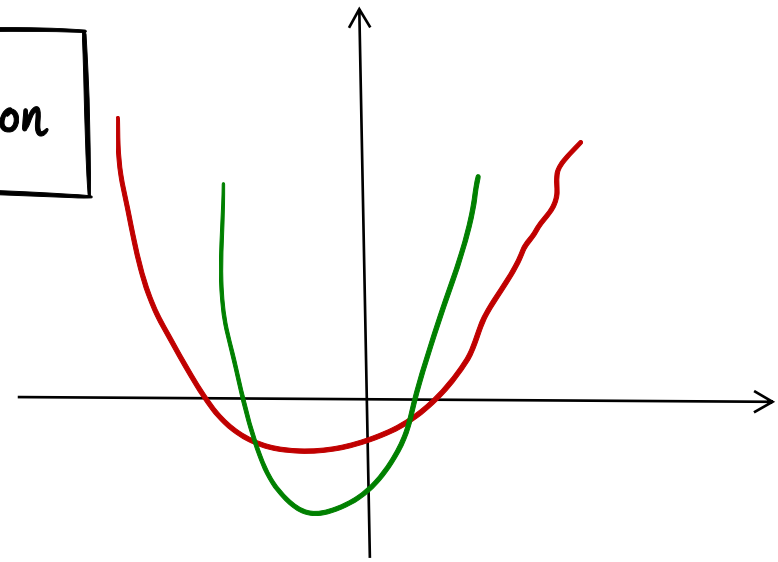
$$x^2 - 7x - 18 = 0$$

$$(x+2)(x-9) = 0$$

$$x+2=0 \quad \text{ie } \underline{x=-2}$$

or

$$x-9=0 \quad \text{ie } \underline{x=9}$$



$$a+b = -7$$

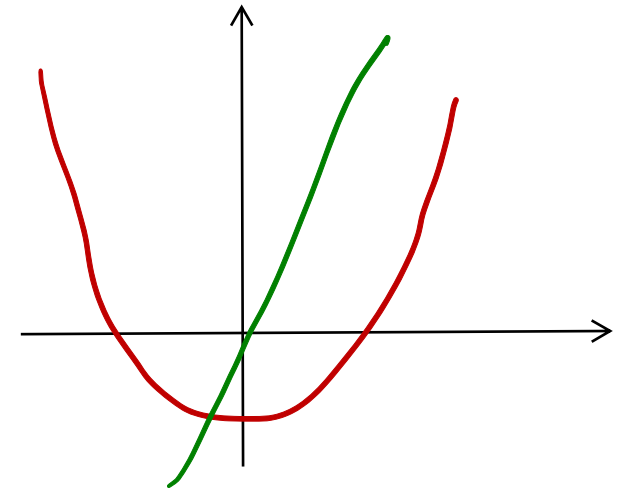
$$ab = -18$$

$$\begin{array}{l} 1 \times 18 \\ 2 \times 9 \end{array}$$

$$-2+9 = 7 \quad \times \quad 3 \times 6$$

$$2+(-9) = -7 \quad \checkmark$$

Solving quadratics by factorization



Find all p for which $3p^2 - 2 = 5p$

$$3p^2 - 5p - 2 = 0$$

$$(3p + a)(p + b) = 3p^2 + ap + 3bp + ab$$
$$= 3p^2 + (a + 3b)p + ab$$

$$(3p + 1)(p - 2) = 0$$

$$a + 3b = -5$$

$$ab = -2$$

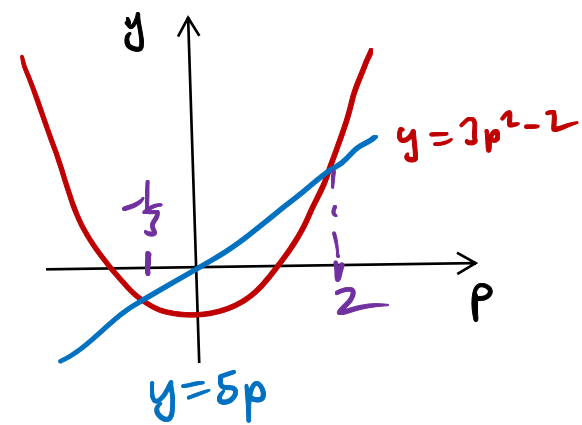
$$3p + 1 = 0 \quad \text{ie } 3p = -1$$
$$\text{ie } p = \underline{\underline{-1/3}}$$

or

$$p - 2 = 0 \quad \text{ie } p = \underline{\underline{2}}$$

a	b	$a + 3b$
2	-1	-1 X
1	-2	-5 ✓
-1	2	
-2	1	

Solving quadratics by factorization



Find all p for which $3p^2 - 2 = 5p$

$$3p^2 - 5p - 2 = 0$$

$$f(p) = 3p^2 - 5p - 2$$

$$f(0) = 3 \times 0^2 - 5 \times 0 - 2 = -2 \neq 0$$

$$f(1) = 3 - 5 - 2 = -4 \neq 0$$

$$f(-1) = 3 + 5 - 2 = 6 \neq 0$$

$$f(2) = 12 - 10 - 2 = 0 \quad \checkmark$$

$p - 2$ is a factor of $f(p)$

$$\underline{3}p^2 - 5p - \underline{2} = (\underline{p} - \underline{2})(\underline{3}p + \underline{1})$$

Factor Theorem

If $f(x)$ is a polynomial
& $f(a) = 0$,
then $x - a$
is a factor of $f(x)$

$$3p^2 - 5p - 2 = 0$$

$$(p - 2)(3p + 1) = 0$$

$$p - 2 = 0 \Leftrightarrow \underline{\underline{p = 2}}$$

OR

$$3p + 1 = 0 \Leftrightarrow 3p = -1$$
$$\Leftrightarrow \underline{\underline{p = -\frac{1}{3}}}$$

Solving quadratics by factorization

Solve $x^2 - 6x + 2 = 0$

$$x^2 - 6x + 2 = (x+a)(x+b)$$

$$(x - \underline{3 + \sqrt{7}})(x - \underline{3 - \sqrt{7}}) = x^2 + (-3 - \sqrt{7})x + (-3 + \sqrt{7})x + (-3 + \sqrt{7})(-3 - \sqrt{7})$$

$$= x^2 - 6x + 9 + 3\sqrt{7} - 3\sqrt{7} - (\sqrt{7})^2$$

$$= x^2 - 6x + 9 - 7$$

$$= x^2 - 6x + 2$$

$x = 3 - \sqrt{7}$, $x = 3 + \sqrt{7}$

$$\begin{cases} ab = 2 \\ a+b = -6 \end{cases}$$

a	b	a+b
1	2	3 X
-1	-2	-3 X