Improving Understanding of Collusion in Intermediate Microeconomics

Evan Moore

Abstract

Standard treatments of collusion in intermediate microeconomics textbooks frequently involve a Cournot duopoly facing linear demand with constant marginal costs of production. These presentations leave students with the misunderstanding that firms jointly behaving like a single-firm monopolist and profit maximising collusion are one and the same. We present a simple and effective way for improving student comprehension of collusion; this exercise results in collusion where the duopolists produce more total output than that of a monopolist while enjoying greater joint profits. The exercise can be used to clarify and lead to a better understanding of collusion and profit maximisation.

JEL classification: A22, D43

1. Introduction

Collusion is broadly defined as an agreement among firms to fix prices or output, usually with the aim of maximising profits. Standard treatments of collusion in intermediate microeconomics textbooks frequently involve a Cournot duopoly facing linear demand with constant marginal costs of production. For example, see:

1. Caroll (2009, p.237) – marginal costs of *k*

2. Perloff (2009, p.459) – marginal costs of $147

3. Pindyck and Rubinfeld (2009, p. 453) – zero marginal costs

4. Frank (2008, p. 415) – zero marginal costs

5. Perloff (2008, p.473) – marginal costs of $147

6. Varian (2007, p. 497) – zero marginal costs

7. Nicholson and Snyder (2006, p. 415) – zero marginal costs

8. Hey (2003, p. 378) – marginal costs of $10

9. Eaton, Eaton and Allen (2002, p. 501) – marginal costs of $40

However, these presentations leave a misunderstanding on the nature of joint profit maximisation. In each of the examples in the textbooks listed above the collusive outcome coincides with each of the duopolists producing one half of the monopolist's output.[1] Unfortunately, if this is the sole presentation of collusion then students frequently equate collusion among firms with jointly behaving as a monopolist.

2. Teaching collusion in intermediate microeconomics

We begin by defining collusion and then pointing out that the goal of colluding is to maximise joint profits. To be more specific, we use Pindyck and Rubinfeld's (2009) definition that when firms collude, '…they coordinate prices and output to maximize joint profits'. We tell students that, like the book, we will use a duopoly with identical cost functions for both firms.[2]

We then present the standard treatment using constant marginal costs of production. We use a linear inverse demand function P=100-Q_1-Q_2 where P is price and Q_i is the output of firm *i*. Each firm has total costs of production TC_i=Q_i. With this total cost function the firms have constant marginal costs MC=1. This results in the firms producing Q_1=Q_2=33 when engaged in Cournot competition. The collusive outcome, which coincides with joint output equal to that of a monopolist, is Q_1=Q_2=24.75. The derivations for these results and those following are in the appendix.[3] Using the outputs as the two strategy choices and profits as the payoffs, we can construct a simple 2x2 normal form game (see Figure 1) revealing the resulting 'prisoners' dilemma' that is commonly associated with Cournot's equilibrium.[4]

Figure 1: Duopolists with marginal costs of 1

		Firm 2	
		24.75	33
Firm 1	24.75	1225.13 1225.13	1361.25 1020.94
	33	1020.94 1361.25	1089 1089

We then inform the class that we are going to change the production costs for the firms. This involves changing the nature of the total costs from linear to quadratic, resulting in TC_i=Q_i^2. The marginal costs are then MC_i=2Q_i. The Cournot equilibrium is Q_1=Q_2=20. The monopolist's profit maximising output is

---

[1] Another textbook presentation involving constant marginal costs is Salvatore (2009, p. 360), which uses four firms with collusion resulting, once again, in the monopoly solution. Besanko and Braeutigam (2008, p.430) provide a different approach with duopolists having differing quadratic total costs, as does Caroll (2009, p.238). However, this overcomplicates the issue and is usually covered in Industrial Organisation textbooks, for example see Waldman and Jensen (2007, p.278) or Pepall *et al.* (2002, p.146).

[2] We use Pindyck and Rubinfeld (2009) in our course.

[3] The interested reader will also find figures containing the reaction functions and isoprofit curves for the exercises as well.

[4] Additionally, we usually provide Figures 1 through 4, with the payoffs, to the students so as not to use too much class time on the profit calculations. In each case the figures are provided after determining the appropriate outputs.

25; splitting this output evenly yields $Q_1=Q_2=12.5$. Using these outputs as the two strategy choices results in the 2x2 normal form game in Figure 2.

Figure 2: Duopolists with marginal costs of $2Q_i$

		Firm 2	
		12.5	20
Firm 1	12.5	781.25 781.25	950 687.5
	20	687.5 950	800 800

The Nash equilibrium is to produce the Cournot output as in Figure 1. However, unlike Figure 1, the joint profits from the Cournot outcome (800+800) exceed those of splitting the monopolist's output (781.25+781.25). We stress to the students that colluding as a monopolist results in lower profits for the firms.

We then use a well known and simple technique to teach collusion when firms face identical demand and cost structures: multiply the slope of the demand curve by the number of firms and solve for the profit maximising output as a monopolist, which is each firms' output.[5] In both of the previous examples this results in inverse demand of $P=100-2Q$. In the first example this results in a profit function of $\Pi=(100-2Q)Q-Q$. The resulting joint profit maximising outputs are $Q_1=Q_2=24.75$, exactly as they are in Figure 1. However, using this technique with the quadratic cost function results in a profit function of $\Pi=(100-2Q)Q-Q^2$ with the collusive outputs of $Q_1=Q_2=16.67$. We build upon Figure 2 by including these outputs as a third strategy choice as illustrated in Figure 3. Figure 3 allows the students to see clearly the profits associated with each output choice.

Figure 3: Duopolists with marginal costs of $2Q_i$ and collusive strategies

			Firm 2	
		12.5	16.66	20
	12.5	781.25 781.25	902.78 729.17	950 687.5
Firm 1	16.66	729.17 902.78	833.33 833.33	866.67 777.77
	20	687.5 950	777.77 866.67	800 800

Using Figure 3 also allows the students to recognise the Nash equilibrium as the Cournot output decision, as in Figure 2.

---

[5] The instructor may opt for the students to determine the collusive outputs by solving the joint profit maximising function for each case, i.e. *Joint* $\Pi=(100-Q_1-Q_2)(Q_1-Q_2)- Q_1-Q_2$ with firms facing $TC_i=Q_i$ and *Joint* $\Pi=(100-Q_1-Q_2)(Q_1-Q_2)-Q_1^2-Q_2^2$ with firms facing $TC_i=Q_i^2$.

Finally, we provide a reduced version of the previous figure, similar to that of Figure 1, that includes only the collusive and Cournot outputs. This is shown in Figure 4.

Figure 4: Prisoners' dilemma with marginal costs of $2Q_i$

		Firm 2	
		16.66	20
Firm 1	16.66	833.33 833.33	866.67 777.77
	20	777.77 866.67	800 800

This figure allows the students to see the Cournot output decisions in the familiar 'prisoners' dilemma' context. More importantly, the exercise as whole reinforces the notion that collusion does not necessarily imply jointly behaving as a monopolist.[6]

Students may ask for an explanation as to why collusion among the duopolists facing quadratic costs does not result in the monopoly level of output. We will provide two possible avenues for explaining this result. The first explanation involves equating the industry marginal revenue with the industry marginal cost to maximise industry profits. Figure 5 presents the industry demand and marginal revenue curves. Additionally, the marginal cost curves for both scenarios, firms facing total costs of $TC_i=Q_i$ and $TC_i=Q_i^2$, are included. The *industry* marginal cost curve for each case is obtained by summing horizontally the firms' marginal cost curves.[7]

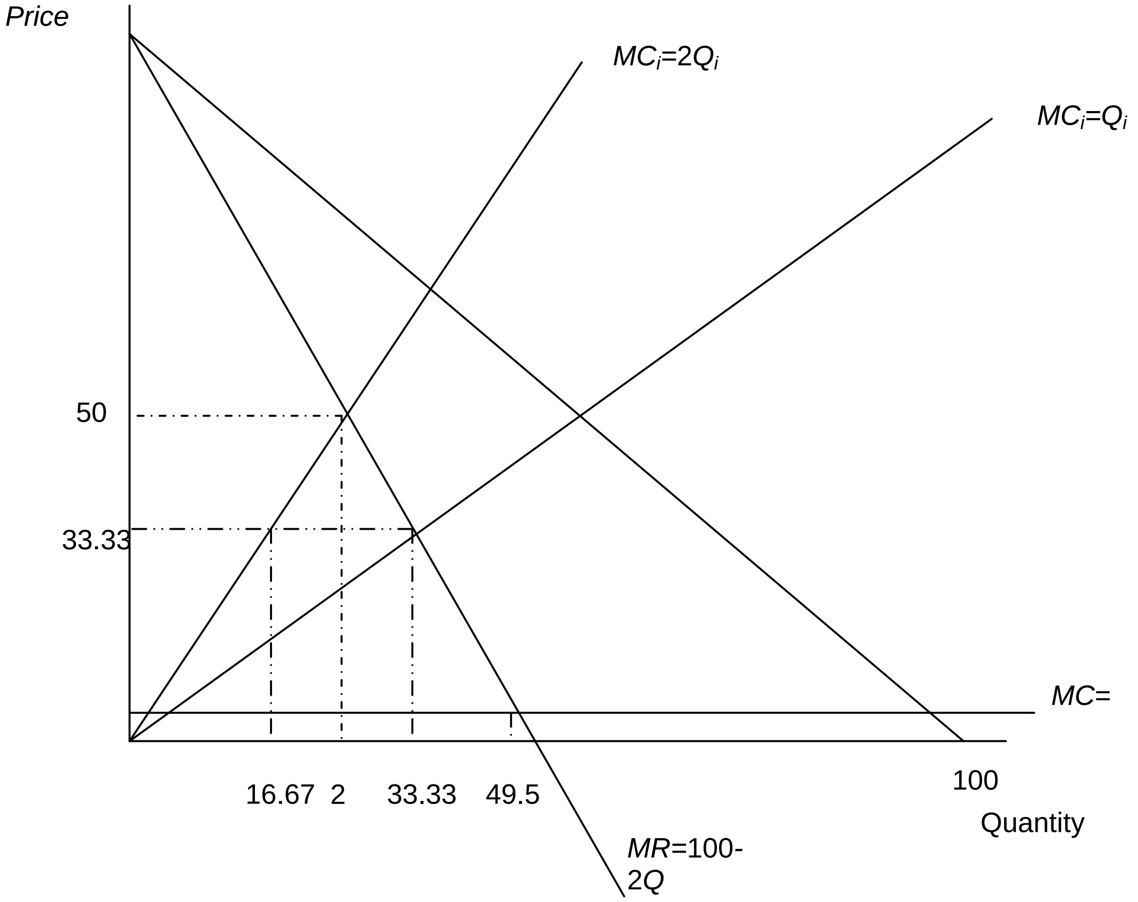
For the firms facing $TC_i=Q_i$, with constant marginal costs of $MC=1$, the industry marginal cost curve is identical to that of any firm. It is this relationship between the industry and firm marginal costs that results in the duopolists jointly producing, in a profit maximising collusive arrangement, the monopolist's output of 49.5 units.

---

[6] The exercise also provides for an opportunity to expound on returns to scale. While not the focus of this paper, we find that drawing the average total cost (ATC) functions and discussing the implications of a constant ATC versus an increasing ATC to be enlightening.

[7] See Salvatore (2009, p.359) or Waldman and Jensen (2007, p.278) for more information on deriving industry marginal cost curves.

Figure 5: Industry marginal costs, demand, and marginal revenue



Now consider firms facing total costs of $TC_i=Q_i^2$. If the market is served by a monopolist then the monopoly's marginal cost curve, $MC=2Q$, and the industry marginal cost curve are identical. This results in the monopolist choosing to producing 25 units to maximise profit. However, a firm's marginal cost curve is not equal to the industry marginal cost curve in the duopoly setting. Each firm faces $MC_i=2Q_i$ while the industry marginal cost is $MC=Q$. For the duopolists the industry marginal costs are lower than those of the monopolist. The duopolists can increase joint output, relative to the monopoly output of 25 units, leading to greater joint profits. The collusive duopolists will jointly produce 33.33 units. Equating the industry wide marginal revenue $MR=100-2Q=33.33$ with each firms marginal cost, $MC_i=2Q_i$, reveals that each firm will produce 16.66 units. This outcome is shown in Figure 5 above.

If the instructor is not interested in using the industry marginal cost diagram then a similar explanation to the one above can be given without using the figure. For the $TC_i=Q_i^2$ scenario, point out that the industry marginal revenue of producing 25 units, the monopolist's output, is $MR=100-2Q=50$. If each firm produces half of the monopolist's output then they will each produce 12.5 units. The marginal cost for each firm is then $MC_i=2Q_i=25$. As the marginal revenue exceeds the marginal cost for each firm, as well as the industry marginal cost, they should increase output to increase profits. The instructor can propose each firm produce 15 units, pointing out that the industry marginal revenue is then $MR=100-2Q=40$ and each firm's marginal cost is $MC_i=2Q_i=30$. Each firm's profits increase from 781.25 to 825. The duopolists will increase output until the joint output equals 33.33. The marginal revenue decreases to

MR =100-2Q = 33.33 and the marginal cost for each firm is MC_i=2Q_i=33.33, as each firm produces 16.66 units. As discussed earlier, this results in each firm earning profits of 833.33.

3. Conclusions

The use of a simple duopoly Cournot model, with quadratic costs, can be very enlightening for undergraduate students when teaching collusion and joint profit maximisation. Unfortunately the standard textbook presentations in principles and intermediate microeconomics usually leave students with the misunderstanding that firms jointly behaving like a single-firm monopolist and collusion are one and the same. We find that the exercise presented in this paper can be used to clarify the issue and lead to a better understanding of collusion and profit maximisation.

References

Besanko, D.A and Braeutigam, R.R. (2005). *Microeconomics* (2nd edn), Hoboken: John Wiley & Sons.

Caroll, T. (2009). *Intermediate Microeconomic Theory* (1st edn), Dubuque: Kendall/Hunt.

Eaton, B.C., Eaton, D.F. and Allen, A.W. (2002). *Microeconomics* (5th edn), Harlow: Prentice Hall.

Frank, R.H. (2008) *Microeconomics and Behavior* (7th ed.), Singapore: McGraw-Hill Education.

Hey, J.D. (2003). *Intermediate Microeconomics,* New York: McGraw Hill Higher Education.

Nicholson, W. and Snyder, C.M. (2006). *Intermediate Microeconomics and its Application* (10th edn), Mason: South-Western College Publishing.

Pepall, L., Richards, D.J. and Norman, G. (2002) *Industrial Organization: Contemporary Theory and Practice* (2nd edn), Mason: South-Western College Publication.

Perloff, J.M. (2008). *Microeconomics: Theory and Applications with Calculus* (1st edn), Harlow: Pearson Education.

Perloff, J.M. (2009). *Microeconomics* (5th edn), Harlow: Pearson Education.

Pindyck, R.S. and Rubinfeld, D.L. (2009). *Microeconomics* (7th edn), Harlow: Prentice Hall.

Salvatore, D. (2009). *Microeconomics: Theory and Applications* (5th edn), Oxford: Oxford University Press.

Varian, H.R. (2005). *Intermediate Microeconomics: A Modern Approach* (7th edn), New York: W. W. Norton.

Waldman, D.E. and Jensen, E.J. (2007). *Industrial Organization: Theory and Practice* (3rd edn), Harlow: Pearson Addison Wesley.

Appendix

$P=100-Q_1-Q_2$ and $TC_i=Q_i$.

For Figure 1

Monopolist output (profit maximising collusion):

$$\Pi=(100-Q)Q-Q$$

$$\frac{d\Pi}{dQ}=100-2Q-1=0$$

$$Q=49.5$$

$$Q_1=Q_2=24.75$$

$$P=100-24.75-24.75=50.5$$

$$\pi_1=\pi_2=50.5*24.75-24.75=1{,}225.13$$

Cournot competition:

$$\Pi_i=(100-Q_i-Q_j)Q_i-Q_i$$

$$\frac{d\Pi_i}{dQ_i}=100-2Q_i-Q_j-1=0$$

$$Q_i=49.5-0.5Q_j$$

$$Q_1=Q_2=33$$

$$P=100-33-33=34$$

$$\pi_1=\pi_2=34*33-33=1{,}089$$

If Q_i = 24.75 and Q_j = 33:

P = 100 – 24.75 – 33 = 42.25

$$\pi_i=42.25*24.75-24.75=1{,}020.94$$

$$\pi_j=42.25*33-33=1{,}361.25$$

$P=100-Q_1-Q_2$ and $TC_i=Q_i^2$.

For Figures 2, 3 and 4

Monopolist output:

$$\Pi=(100-Q)Q-Q^2$$

$$\frac{d\Pi}{dQ}=100-2Q-2Q=0$$

$$Q=25$$

$$Q_1=Q_2=12.5$$

$$P=100-12.5-12.5=75$$

$$\pi_1=\pi_2=75*12.5-12.5^2=781.25$$

Cournot competition:

$$\Pi_i=(100-Q_i-Q_j)Q_i-Q_i^2$$

$$\frac{d\Pi_i}{dQ_i}=100-2Q_i-Q_j-2Q_i=0$$

$$Q_i=25-0.25Q_j$$

$$Q_1=Q_2=20$$

$$P=100-20-20=60$$

$$\pi_1=\pi_2=60*20-20^2=800$$

If Q_i = 12. 5 and Q_j = 20:

P = 100 – 12.5 – 20 = 67.5

$$\pi_i=67.5*12.5-12.5^2=687.5$$

$$\pi_j=67.5*20-20^2=950$$

Profit maximising collusion:

$$\Pi_i=(100-2Q_i)Q_i-Q_i{}^2$$

$$\frac{d\Pi_i}{dQ_i}=100-4Q_i-2Q_i=0$$

$$Q_i=16.67$$

$$P=100-16.67-16.67=66.66$$

$$\pi_1=\pi_2=66.66*16.67-16.67^2=833.33$$

If Q_i = 12. 5 and Q_j = 16.66:

P = 100 – 12.5 – 16.66 = 70.83

$$\pi_i=70.83*12.5-12.5^2=729.17$$

$$\pi_j=70.83*16.66-16.66^2=902.78$$

If Q_i = 16.66 and Q_j = 20:

P = 100 – 16.66 – 20 = 63.33

$$\pi_i=63.33*16.66-16.66^2=777.77$$

$$\pi_j=63.33*20-20^2=866.67$$

Figures containing reaction functions and isoprofit curves

Figures A1, A2, and A3 pertain to the duopoly scenario with each firm having total costs of production $TC_i=Q_i$.

Figure A1: Reaction functions – firms facing $TC_i=Q_i$

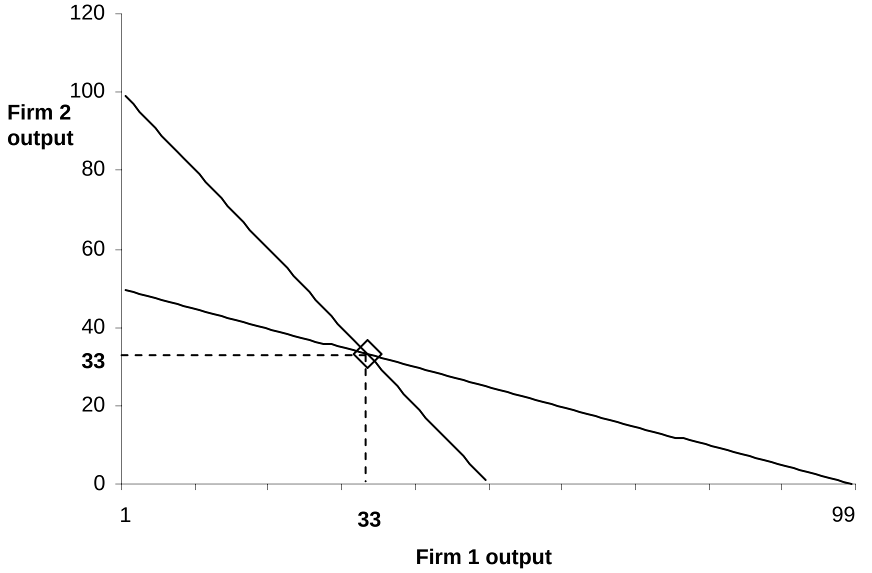


Figure A2: Isoprofit curves – firms facing $TC_i=Q_i$

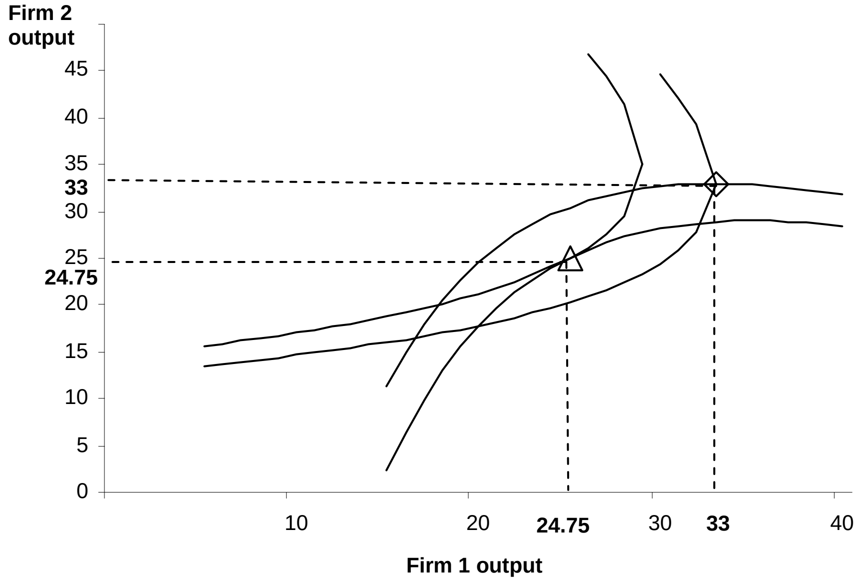
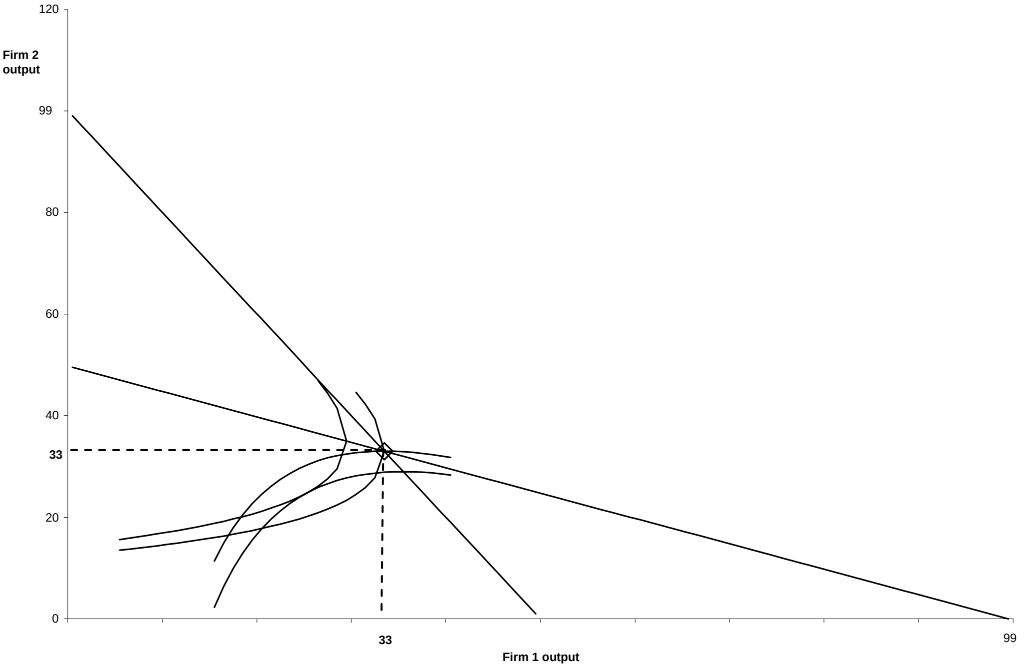


Figure A1 presents the reaction functions $Q_i=49.5-0.5Q_j$ for the duopolists. Note that the intersection at $Q_1=Q_2=33$ is the Cournot equilibrium outputs (indicated by ◊ in the figure). Figure A2 presents the isoprofit curves. The curves indicating profits of 1089 for each firm intersect at $Q_1=Q_2=33$, the Cournot equilibrium (again indicated by ◊). Joint profits are maximised at the tangency of the isoprofit curves at

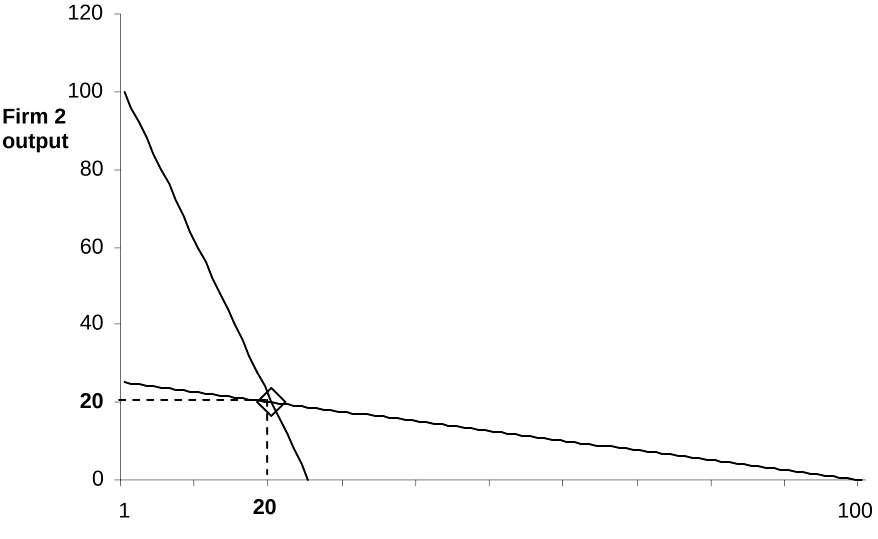
$Q_1=Q_2=24.75$ (this is indicated by Δ). This collusive outcome results in each firm earning profits of 1225.125. Figure A3 combines Figures A1 and A2 revealing that the intersection of the reaction functions and the Cournot equilibrium isoprofit curves coincides with the Cournot equilibrium outputs.

Figure A3: Reaction functions and isoprofit curves - firms facing $TC_i=Q_i$



Figures A4, A5, and A6 pertain to the duopoly scenario with each firm having total costs of production $TC_i=Q_i^2$. The reaction functions in Figure A4 are $Q_i=25-0.25Q_j$. The Cournot outcome, $Q_1=Q_2=20$, is indicated by ◊.

Figure A4: Reaction functions – firms facing $TC_i=Q_i^2$



Unlike the previous scenario with constant marginal costs of production, the collusive outcome does not coincide with each firm producing half of the monopolist's level of output. The isoprofit curves for each firm, as shown in Figure A5, represent the profits from collusion, i.e. 833.33, from Cournot competition, i.e. 800, and from producing half of the monopolist's level of output, i.e. 781.25. The Cournot outcome, Q_1=Q_2=20, is indicated by ◊ and the collusive outcome, Q_1=Q_2=24.75, by Δ.

Figure A6 combines Figures A4 and A5 revealing that the intersection of the reaction functions and the Cournot equilibrium isoprofit curves coincides with the Cournot equilibrium outputs.

Figure A5: Isoprofit curves – firms facing $TC_i=Q_i^2$

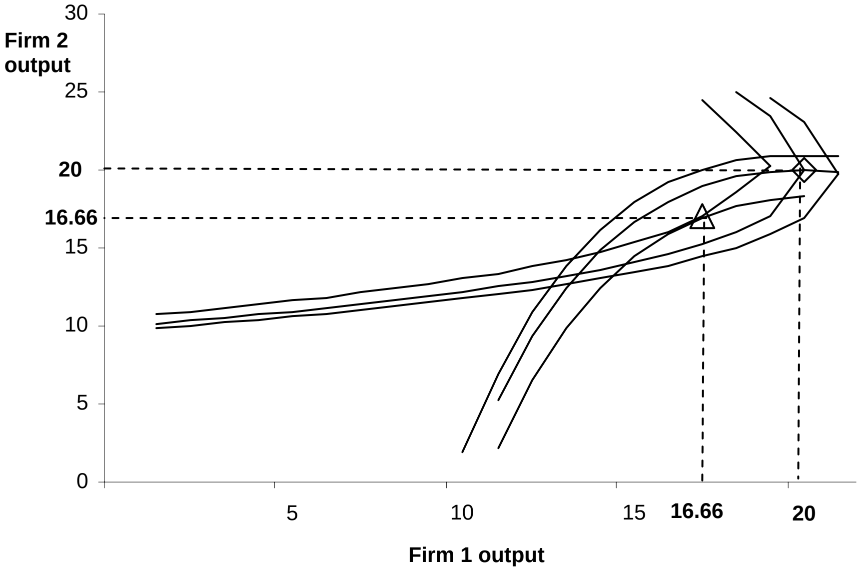
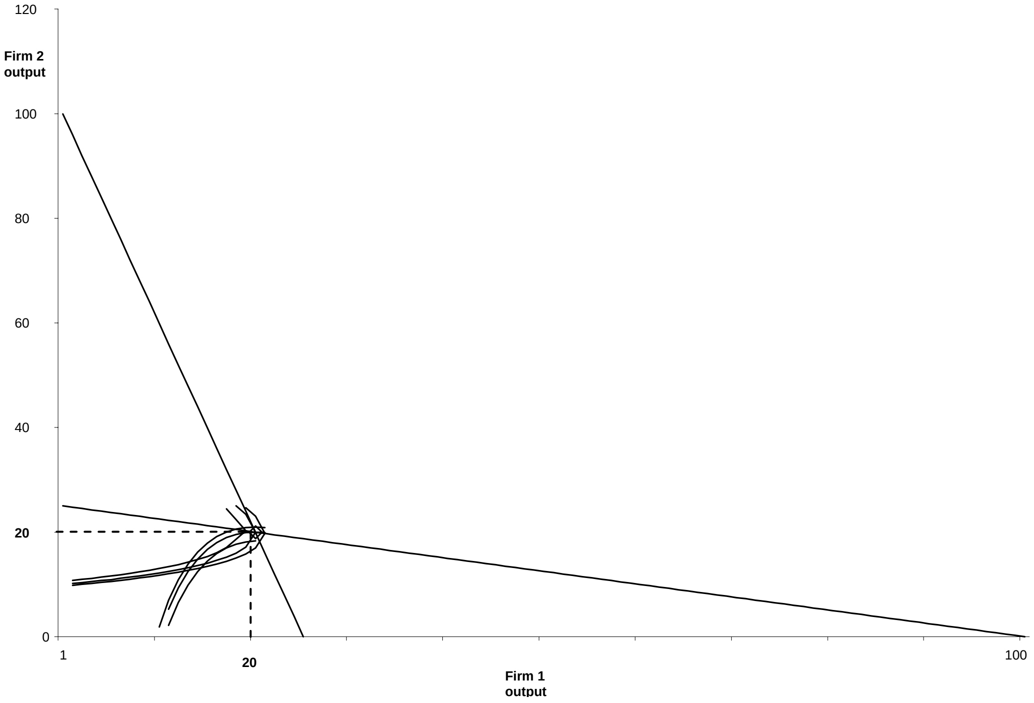


Figure A6: Reaction functions and isoprofit curves - firms facing $TC_i=Q_i^2$



Author Biography

Evan Moore is an Associate Professor and Head of the Department of Economics at Auburn University at Montgomery. He also serves as an editor of *The Southern Business and Economic Journal*. He holds a Ph.D. from Virginia Polytechnic Institute and State University (Virginia Tech). His teaching interests include microeconomics and industrial organization. His primary research interests include industrial organization, particularly firm strategy, and experimental economics.

Contact details

Evan Moore
Auburn University at Montgomery
Department of Economics
P.O. Box 244023
Montgomery, Alabama, USA 36124-4023
Tel: 334-244-3364
Fax: 334-244-3792
Email: emoore1@aum.edu