

Does income constrain household spending?

1. Introduction

Households are the single most important consumers of a nation's goods and services. Therefore, it is important to understand the influences on their spending decisions. In this case study we apply mathematical techniques to analyse the *long-run* relationship in the UK between income and consumption. We ask whether income constrains the household sector's spending over the longer run. In the case study 'Do households smooth their consumption?' the responsiveness of consumption to income in the *short-run* is considered.

2. UK Consumption and income

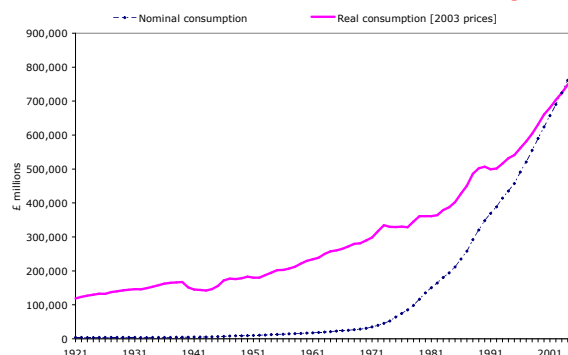
Figures from the Office for National Statistics show that consumption by UK households in 2006 amounted to £826.6 billion.¹ Charles Feinstein estimates that household consumption in 1921, when the current borders of the UK were established, was £4.3 billion.² However, since the aggregate price level of goods and services has increased over this period we cannot directly infer changes in the *volume* of consumption. To do this we use constant price estimates of household consumption.

Constant price series are created for categories of household spending. Each series is constructed by stripping out the impact of price changes and fixing on the price level in a particular year, known as the base year. These series are aggregated to form a constant price series for total household expenditure.

Chart 1 shows household expenditure in the UK from 1921 at current prices (nominal consumption) and at constant 2003 prices (real consumption). The constant price or real consumption numbers are higher than their current

price or nominal counterparts in the years before 2003. This is because the nominal numbers have been scaled up to reflect the higher consumer price level of 2003.

Chart 1: Nominal and real consumption



Sources: (i) Feinstein, C.H., 1972 (figures from 1870-1955); (ii) National Statistics (figures from 1955)

After 2003, the nominal series is scaled down to reflect the lower consumer price level in 2003. In 2003 the nominal and real values are equivalent.

Consumption at constant 2003 prices in 2006 is estimated at £774.8 billion compared with £119.0 billion in 1921. These numbers can be used to establish an annual compound growth rate for real consumption. It is calculated by subtracting 1 from the result of taking the *n*th root of the relative value in the final year, *V*, to that in the first year, *A*, where *n* is the number of years considered.

$$(1) \quad g = \left(\frac{V}{A}\right)^{\frac{1}{n}} - 1$$

By entering our numbers we find that the volume of consumption grew by 2.2% per annum between 1921 and 2006.

$$(2) \quad g = \left(\frac{774.82}{119.00}\right)^{\frac{1}{85}} - 1 = 0.0223$$

We can also control for the growth in the UK's population which rose from 44 million in 1921 to 61 million in 2006. At constant 2003 prices, consumption per capita rose from £2,700 in 1921 to £12,784 in 2006. This is an annual compound growth rate of 1.8%.

¹ National Statistics Online can be accessed at <http://www.statistics.gov.uk/>

² Feinstein, C.H., 1972. National Income, Expenditure and Output of the UK 1855-1965. Cambridge: Cambridge University Press.

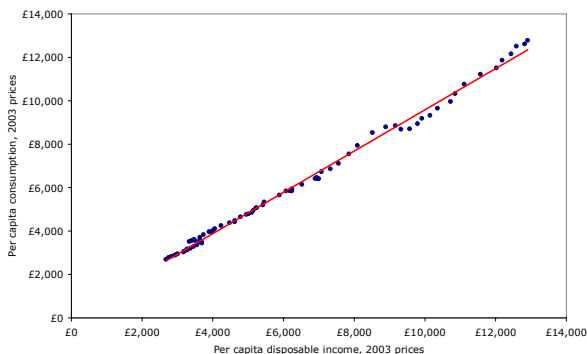
$$(3) \quad g = \left(\frac{12,783.84}{2,700.24} \right)^{\frac{1}{85}} - 1 = 0.01846$$

Disposable income reflects wages after taxes, interest payments and pension contributions. Between 1921 and 2006 disposable income per capita at constant 2003 prices rose from £2,671 to £12,901. This is an annual compound growth rate of 1.9% per annum, virtually the same as for consumption.

$$(4) \quad g = \left(\frac{12,901.47}{2,670.83} \right)^{\frac{1}{85}} - 1 = 0.01870$$

Chart 2 is a scatter plot of combinations of real per capita consumption (C) and real per capita disposable income (Y) since 1921.³ We have included a *line of best fit* which attempts to find the equation of a line closest to our data.⁴ The vertical distance between the line and the observations measures our model's error.

Chart 2: Real per capita consumption and income



Sources: As Chart 1

The equation of the straight line has an intercept of 87.883 and a slope coefficient of 0.9496 such that

$$(5) \quad C = 87.883 + 0.9496 Y$$

The intercept is known as *autonomous consumption*. It captures that part of spending independent of disposable income. Of particular interest is the slope. This captures the *marginal propensity to consume* [MPC]. It is the *change* in consumption following an incremental change in income, for

³ There are no observations from 1939 to 1945.

⁴ This can be done in Excel by using the Add Trendline option in the Chart menu. Go to the Options menu to display the equation of the line.

example an increase of £1. A MPC of 0.95 means that an increase in real per capita disposable income of £1 sees real per capita consumption increase by 95p.

By modelling a linear consumption-income relationship we are making several assumptions. To see this, we write our linear per capita consumption function as

$$(6) \quad C = \alpha + \beta Y$$

α is *autonomous consumption* and β the MPC.

With this linear consumption function, the *MPC is constant* (β). Therefore, regardless of the initial level of disposable income, a given incremental increase in income will always induce the same increase in the level of consumption.

The proportion of income spent is known as the *average propensity to consume* (APC). The APC can be expressed as

$$(7) \quad APC = \frac{C}{Y}$$

Substituting in for per capita consumption we find

$$(8) \quad APC = \frac{\alpha + \beta Y}{Y} = \frac{\alpha}{Y} + \beta$$

Our linear consumption function means that, unless α is zero, the *APC falls as income rises*. Specifically, as income rises α/Y decreases and the proportion of income spent converges on β , the marginal propensity to consume. If α is zero then the APC is equal to the MPC.

Our linear consumption function also has implications for the value of the *income elasticity of consumption* (ε_Y). This measures the percentage change in consumption following a 1% change in disposable income. By analysing percentage changes we avoid the problem of the MPC whereby a given change in the level of income becomes proportionately smaller as income rises.

The income elasticity of consumption can be written as

$$(9) \quad \varepsilon_Y = \frac{\frac{\Delta C}{C} * 100}{\frac{\Delta Y}{Y} * 100}$$

ΔC and ΔY are short-hand for the change in consumption and income respectively. The numerator of (9) is the percentage change in consumption, while the denominator is the percentage change in income. This can be rearranged such that

$$(10) \varepsilon_Y = \frac{\Delta C}{\Delta Y} * \frac{Y}{C}$$

The first term on the RHS of the identity is the gradient of the consumption function. The second term is the inverse of the APC.

With our linear per capita consumption function the gradient is β , while the inverse of the APC is

$$(11) \frac{Y}{\alpha + \beta Y}$$

Hence, the income elasticity of consumption is

$$(12) \varepsilon_Y = \frac{\beta Y}{\alpha + \beta Y}$$

As disposable income increases, the income elasticity converges on 1 because the value of α becomes increasingly small. If α is zero then the income elasticity of consumption equals 1.

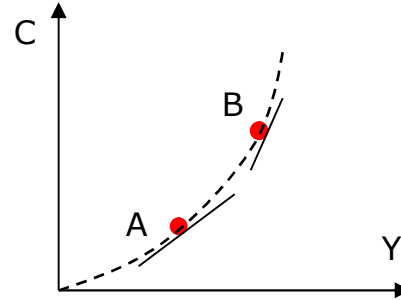
A restrictive implication of the linear consumption function is the MPC being constant. In practice most models that economists estimate are non-linear. A frequently used non-linear form is the *power function*. When applied to consumption this may be expressed as

$$(12) C = \beta Y^\eta$$

With the power function the *variable base* Y (per capita income) is raised to a *fixed power* η . β is a *scalar* which simply moves the values of Y^η up or down.

If η is positive and non-zero, as expected, the consumption function passes through the origin. Interestingly, if the power η equals 1 then the consumption-income relationship is linear. However, for all other values the consumption function is non-linear and its gradient varies. In Figure 1 we show the graph of the function where $\eta > 1$ and the gradient of the consumption curve grows steeper.

Figure 1: Power consumption function



For small changes in income, the gradient, $\Delta C/\Delta Y$, is approximated by *tangents* to the consumption curve. A tangent is a straight line which passes through points like A and B on the curve and which just touches the curve. The slope of the graph of a function is called the *derivative of the function*. In the case of a non-linear function there is a different derivative at each point on the graph.

By finding the slope of the graph at particular points we define a slope function commonly referred to as a *derived function*. This function can be found through the process of *differentiation*. With a power function this involves bringing the power down to the front and subtracting 1 from the power. The derived function, dC/dY is

$$(13) \frac{dC}{dY} = \eta \beta Y^{\eta-1}$$

We can use the derived function from our power consumption function to show that the power function results in a *constant income elasticity of consumption*. Substituting the derived function (13) into our expression for the income elasticity of consumption (10) we have

$$(14) \varepsilon_Y = \eta \beta Y^{\eta-1} * \frac{1}{APC}$$

The APC is found to be

$$(15) APC = \frac{\beta Y^\eta}{Y} = \beta Y^{\eta-1} = \beta Y^{\eta-1}$$

The relationship between the APC and income is dependent on the power, η . When $\eta < 1$ the APC decreases as disposable income increases, while in the case when $\eta > 1$ the APC increases.

When η is 1 the APC is constant and equal to β .

Substituting for the APC in (14) we find that *the income elasticity is constant and equal to the power, η .*

$$(16) \quad \varepsilon_Y = \frac{\eta\beta Y^{\eta-1}}{\beta Y^{\eta-1}} = \eta$$

3. Long-run consumption function

The power function is popular in modelling relationships because taking logs gives a *log-linear relationship*. This makes the estimation of the unknown power more straightforward.⁵ Taking the log of both sides of our power consumption function we get

$$(17) \quad \log_b C = \log_b[\beta Y^\eta]$$

To arrive at a log-linear specification we employ two rules of logarithms. Firstly, we employ the *multiplication rule* whereby a multiplication inside the log can be turned into addition outside of the log. Hence,

$$(18) \quad \log_b C = \log_b \beta + \log_b Y^\eta$$

Secondly, we employ the *power rule* such that a power inside a log can be moved to the front of the log. Consequently, the coefficient at the front of the income variable is the income elasticity of consumption.

$$(19) \quad \log_b C = \log_b \beta + \eta \log_b Y$$

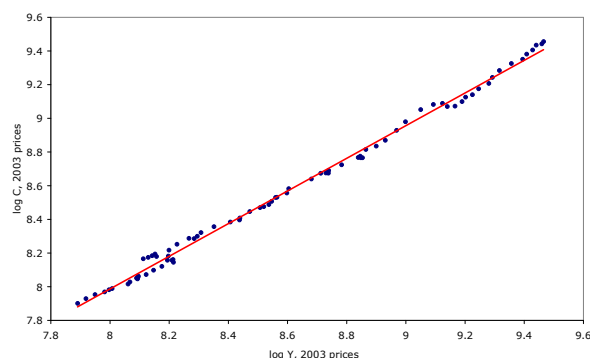
On our calculators *log* represents the *common logarithm*. This is the logarithm with base 10. Chart 3 is a scatter chart of combinations of the common log values of real per capita consumption and income in the UK since 1921. It incorporates a line of best fit. This is our model of the long-run relationship between consumption and income. The equation of the line has an intercept of 0.227 and a slope coefficient of 0.97.

$$(20) \quad \log_{10} C = 0.227 + 0.97 \log_{10} Y$$

Our main interest is the coefficient on income. This is the *long-run* income

elasticity of consumption since the long sample period means that we are capturing the effect of permanent income changes on consumption. A permanent increase of 1% in households' real per capita income is found to increase their real per capita consumption by 0.97%.

Chart 3: Log-linear consumption function



Sources: As Chart 1

The estimation infers that the household sector's consumption and income grow together in the long-run. This supports the hypothesis that households satisfy their life-time or inter-temporal budget constraint. In other words, while our spending and income may diverge from time to time, over our life times they are pretty much equivalent.

Tasks

The following table displays data from National Statistics on consumption and disposable income at constant 2003 prices in £ millions.

	Consumption	Disposable income
1976	£330,156	£348,733
1986	£427,800	£444,924
1996	£561,758	£602,417
2006	£774,816	£781,945

- (i) Calculate the compound growth rates from 1976, 1986 and 1996 for real consumption and real income.
- (ii) Use the growth rates to calculate approximations of the income elasticity of consumption.

⁵ The logarithm of a number y with respect to a base b is the exponent to which we have to raise b to obtain y . Therefore, $\log_b y = x$ means $b^x = y$.