Hypothesis Testing (2)

Lecture 8

Hypothesis Testing (2)

- So far we have looked at hypothesis testing about a single population parameter
  - H_0: μ=5000
- Now we look at testing hypotheses about differences between two population parameters
  - E.g. are women paid less than men?

Formal Layout of a Problem

1. Specify the null and alternative hypothesis
2. Choose significance level, e.g. 5%
3. Look up critical value from z or t-tables
4. Calculate the test statistic
5. Decision: reject or do not reject H_0

Testing the Difference of Two Means

- To test whether two samples are drawn from populations with the same mean

H_0: $\mu_1 = \mu_2$ or H_0: $\mu_1 - \mu_2 = 0$
H_1: $\mu_1 \neq \mu_2$ or H_0: $\mu_1 - \mu_2 \neq 0$

- The test statistic is

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Example: Car Company has Two Factories is Output the Same?

Average daily output for 30 days	420	408
Standard dev of daily output	25	20

- H_0: $\mu_1 = \mu_2$ or $\mu_1 - \mu_2 = 0$
- H_1 : $\mu_1 - \mu_2 \neq 0$
- Significance level of 1% implies $z^*_{0.005} = 2.57$

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(420 - 408) - (0)}{\sqrt{\frac{25^2}{30} + \frac{20^2}{30}}} = 2.05$$

- $z < z^*$ so it falls in the non-rejection region and therefore we do not reject the null hypothesis.

- There does not seem to be enough evidence to reject the claim that output is the same in each factory

Testing the Difference of Two Proportions

- To test whether two sample proportions are equal

H_0: $\pi_1 = \pi_2$ or H_0: $\pi_1 - \pi_2 = 0$
H_1: $\pi_1 \neq \pi_2$ or H_1: $\pi_1 - \pi_2 \neq 0$

- The test statistic is

$$z = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\sqrt{\frac{\hat{\pi}(1-\hat{\pi})}{n_1} + \frac{\hat{\pi}(1-\hat{\pi})}{n_2}}}$$

$$\hat{\pi} = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

Example: Customer Satisfaction

Are Customers Equally Satisfied with Different Tours?

proportion who say they are satisfied	45/75	48/90

- H_0: $\pi_1 - \pi_2 = 0$
- H_1: $\pi_1 - \pi_2 \neq 0$
- Significance level of 5%, $z^*_{0.025} = 1.96$

$$\hat{\pi} = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{75 * 0.6 + 90 * 0.533}{75 + 90} = 0.564$$

$$z = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\sqrt{\frac{\hat{\pi}(1-\hat{\pi})}{n_1} + \frac{\hat{\pi}(1-\hat{\pi})}{n_2}}}$$

$$= \frac{(0.6 - 0.533) - (0)}{\sqrt{\frac{0.564(1-0.564)}{75} + \frac{0.564(1-0.564)}{90}}} = 0.86$$

- $z < z^*$ so we do not reject the null hypothesis

Small Samples: Testing the Difference of Two Means

- The test statistic is

$$t = \frac{\left(\bar{x}_1 - \bar{x}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{\frac{S^2}{n_1} + \frac{S^2}{n_2}}} \sim t(n_1 + n_2 - 2)$$

- where S^2 is the pooled variance

$$S^2 = \frac{\left(n_1 - 1\right)s_1^2 + \left(n_2 - 1\right)s_2^2}{n_1 + n_2 - 2}$$

Independent and Dependent Samples

- So far assumed our samples are drawn independently
- Often we have dependent samples (e.g. before and after tests, or scores on macro and micro exams)
- We use a different approach to use the info. that the data comes from the same observation

Example: Paired Samples

Worker	1	2	3	4	5	6	7	8	9	10
Before	21	24	23	25	28	17	24	22	24	27
After	23	27	24	28	29	21	24	25	26	28
Improvement	2	3	1	3	1	4	0	3	2	1

$$\bar{x}_B = 23.5, s_B = 3.10, n = 10$$

$$\bar{x}_A = 25.5, s_A = 2.55, n = 10$$

$$\bar{x}_{\text{Improvement}} = 2.00, s_{\text{Improvement}} = 1.247, n = 10$$

$$H_0 : \mu_{\text{improvement}} = 0$$

$$H_A : \mu_{\text{improvement}} > 0$$

$$t^*_{0.05,9} = 1.833$$

$$t = \frac{2.0 - 0}{\sqrt{\frac{1.247^2}{10}}} = 5.07$$

- t>t* so we reject the null hypothesis and conclude that training has improved worker productivity

Summary

- The principles are the same for all tests: calculate the test statistic and see if it falls into the rejection region
- The formula for the test statistic depends upon the problem (mean, proportion, etc)
- The rejection region varies, depending upon whether it is a one or two tailed test
- For large samples we can always use a z test
- If n is small we can still use z test if the population variance is known
- Otherwise use a t (and pool variances)