BS2551   Money Banking and Finance

Ever since the work of Sharpe (1964) the Capital Asset Pricing Model (CAPM) is the most commonly used model to price assets.

The CAPM implies that the risk premium that any risky asset i must offer (in order to be held by the investor) is equal to the price times the quantity of *systematic* risk. *Systematic* risk is risk that cannot be diversified away, which is also known as undiversifiable risk.

The CAPM is defined as

$$E\left[R_i\right] = R_f + \left(E\left[R_m - R_f\right]\right)\beta_i$$

Where the expected return on the market premium represents the price of risk and beta represents the quantity of risk.

The beta parameter for a portfolio of N risky assets is the sum of the individual beta coefficients (B_p), weighted by the percentage of investments (w_i).

$$\beta_p = \sum_{i=1}^{N} w_i \beta_i$$

B > 1, "aggressive investment", for instance a stock that has a beta of 2 on average is twice as risky as the market portfolio.

0 < B < 1, "defensive investment", for instance a stock that has a beta of 0.5 on average is half as risky as the market portfolio.

A portfolio consisting of 60% from stock 1 and 40% of stock 2 has systematic risk equal to:

$B_p = 0.6(2) + 0.4(0.5) = 1.4$, "aggressive"

If $E[R_m] = 0.15$, $R_f = 0.1$

The expected portfolio return is:

$0.1 + (0.15-0.1)1.4 = 0.17 > E[R_m)$.

Portfolio Strategy: To achieve higher return, buy high beta stocks, which are more risky but yield higher potential returns (fat tail distributions, skewness, Gregoriou et al (2004)).

Prior to the CAPM, it was believed that the expected return from an asset varied with total risk, i.e, the variance or standard deviation of returns. The key behind the CAPM is that the total risk of an individual asset is irrelevant in asset pricing. What matters is the systematic risk because the remainder of the risk is diversified away by dynamic hedging investment strategies.

Suppose we have the following two stocks:

Stock A	Variance = 40%	Beta = 0.5
Stock B	Variance = 20%	Beta = 1.5

Stock A has greater total risk, as measured by the variance of returns.

Stock B has greater systematic risk as measured by beta, and therefore according to the CAPM offers greater expected return.

Empirical Estimation of the CAPM

The CAPM makes an implicit assumption that a market portfolio exists. The market portfolio in principle consists of all the assets available to investors, which can be used to diversify away risk. Once all risk has been diversified away what remains is systematic risk, which is captured by beta. Beta then determines the expected return of the asset.

A potential shortcoming of the CAPM is that the "true" market portfolio is unobservable, Roll (1977). The typical proxies for M e.g the FTSE All Share in the UK, are not the "market", since the

total market available to investors includes many additional stocks in the UK, and thousands more in foreign markets. Moreover, the market does not only include equities, but bonds, property, commodities and human capital built up by education and industrial experience.

Extensions of the CAPM

Despite the fact that virtually every assumption underlying the CAPM is violated in reality, a number of studies show that it is possible to relax some of the unrealistic assumptions of the model without drastically affecting the empirical predictions of the model.

No risk free asset: Zero-beta CAPM

Black (1972) showed that the major results of the CAPM do not require the existence of a risk-free asset that has constant returns in every state of nature.

Without access to a risk free asset, investors instead use a zero-beta portfolio, i.e. a portfolio of risky assets with zero covariance with the market portfolio.

According to the Black (1972) zero-beta CAPM the expected return on risky asset i is given by:

$$E\left[R_i\right] = E\left[R_z\right] + \left(E\left[R_m\right] - E\left[R_z\right]\right)\beta_i$$

where $E\left[R_z\right]$ is the expected return on the zero-beta portfolio Z: Portfolio Z is defined as the portfolio that has the minimum variance of all portfolios uncorrelated with M.

The zero-beta CAPM implies that beta is still the correct measure of systematic risk and the model still has a linear specification.

Non Marketable Assets

For some assets the transaction costs are infinite (e.g. block trades and dynamic hedging in option markets), or by law or regulation an asset does not have a market value. For example human capital is non marketable because it cannot be bought or sold in the marketplace.

This in effect introduces a non-diversifiable risky asset in the portfolio of every investor (his/her human capital). As Mayers (1972) points out there are two important implications of non marketable risky assets:

i) Individual investors will hold various portfolios of risky assets since their human capital has differing amounts of risk.

ii) There is an objectively determined market price of risk that is independent of individuals attitudes to risk.

CAPM in continuous time (ICAPM)

Merton (1973) derived a version of the CAPM that relaxes the single time period assumption. In this intertemporal CAPM (ICAPM), trading takes place continuously.

In contrast to the standard CAPM, in the context of ICAPM investors are concerned not only with end of period wealth, but also with the opportunities of consumption and investment throughout the life of the stock.

Therefore when choosing a portfolio at time period t-1, ICAPM investors consider how their wealth at time period t might vary with future state variables, including income from labour, price of

consumption goods and the nature of investment opportunities.

The ICAPM continuous time framework suggests that we have to make further assumptions concerning the evolution of the risk-free rate over time.

If the risk free rate of interest is non stochastic (i.e. fixed) then the ICAPM is a replicate of the standard CAPM model.

If the risk free rate is non stochastic (i.e. allowed to vary over time) then the ICAPM becomes:

$$E\left[R_i\right] = r_f + \left(E\left[R_m\right] - r_f\right)\gamma_1 + \left(E\left[R_n\right] - r_f\right)\gamma_2$$

Since investors are exposed to the risk of unfavourable risk due to the stochastic interest rate, they to hedge their position by including portfolio N in the set of funds that they consider for portfolio

formation (in addition to the portfolio M and the risk free asset). Thus, the model exhibits three-fund separation.

Heterogeneous Expectations

If investors do not have the same information set about the distribution of future returns, they will perceive different opportunity sets and choose different portfolios.

With heterogeneous expectations, M is not necessarily efficient and hence the CAPM is non-testable.