INTRODUCTION TO
NATURAL RESOURCE ECONOMICS

Lecture 1

Natural resource exploitation: basic concepts

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1 Introduction

Natural resources can be defined as natural assets or endowments from which we derive value (utility). A broad definition would include environmental assets such as wilderness which, while they can be destroyed by human activity, do not have to be consumed in order to have value. We could also include the environment as a provider of "ecosystem services" or as an assimilator of waste. Here we adopt a narrower definition, however, and focus on natural resources that must be *extracted* or *harvested* in order to have value, either directly or as inputs into production processes.

A distinction is usually made between *renewable* and *non-renewable* natural resources. Renewable resources are capable of natural replenishment or *growth* on some economically meaningful timescale, for example fish stocks or forests. Non-renewable resources are those that are incapable of any significant growth on such a timescale, for example fossil fuels (coal, oil and gas) metal ores and diamonds. Stocks of non-renewable resources are therefore essentially fixed and are necessarily depleted through extraction.

In general, because natural resources are stocks they are extracted or harvested over more than one period. The efficient and optimal use of natural resources therefore has an inherent *time* dimension.

2 Capital theory

We can think of natural resources as a form of capital (hence *natural capital*). Holding a natural resource stock is analogous to holding a (manufactured) capital asset or a financial asset. The return from holding the asset should be *at least* as high as would be expected if the current value of the asset were invested elsewhere.

From capital theory we have the **arbitrage equation** for an asset, which we can write as

$$y(t) = rp(t) - \dot{p}, \tag{1}$$

where $y(t)$ is the yield from the asset at time t, $p(t)$ is the price (value) of the asset at time t, $\dot{p}$ is the rate of increase of p over time (i.e., $dp(t)/dt$) and r is the rate of return on an appropriate alternative asset (often referred to as the *numeraire* asset), for example, the interest rate on a cash investment. If $\dot{p}$ is positive, the asset is *appreciating* in value, whereas if $\dot{p}$ is negative the asset is *depreciating* in value, as would usually be the case for manufactured capital (for a cash investment, $\dot{p}$ is zero). The arbitrage equation (also known as the "short run equation of yield") gives the condition for holding an asset: it states that the yield should be (at least) equal to the return from the numeraire asset, *less* any appreciation in the asset's value or *plus* any depreciation in its value. If the arbitrage equation is not satisfied, it would be better to sell the asset and invest in the numeraire asset instead.

A stock of a non-renewable resource is said to be *sterile*. This means that it does not exhibit any intrinsic growth and does not (therefore) produce a yield. In this case, $y(t)$ is equal to zero. If we then rearrange (1) we get

$$\frac{\dot{p}}{p(t)} = r. \tag{2}$$

This is a version of a rule for the efficient extraction of a non-renewable resource stock known as **Hotelling's Rule**. Equation (2) states that the value of the non-renewable resource stock must increase at a rate equal to the rate of return on the numeraire asset.

A renewable resource stock, on the other hand, is productive through natural growth and is therefore capable of producing a yield. Assuming, for the sake of argument, that $\dot{p} = 0$, equation (1) can be rearranged to give

$$\frac{y(t)}{p(t)} = r. \tag{3}$$

Equation (3) requires that the yield (or *rent*) from the resource is sufficient to provide an "internal rate of return" $(y(t)/p(t))$ at least as great as the interest rate r. Given one or two assumptions, this is equivalent to requiring that the *growth rate* of the resource is equated with the interest rate.

3 Discounting

In general, individuals exhibit *positive time preferences* over consumption. Thus, consumption (money) now is preferred to consumption (money) deferred to a later date. In capital markets, interest rates are the prices that induce individuals to save, i.e., to forego consumption now in favour of consumption later. Market interest rates are affected by risk, inflation, taxation, etc., but we can think of an underlying social discount rate or "pure" social rate of time preference which a social planner might use in decision making. We will henceforth denote this rate as r. Note that since the discount rate and the interest rate are essentially the same, the discount rate reflects the *opportunity cost* of investment (saving).[1]

If, from (2), we have

$$\dot{p} = rp(t),$$

then it follows that

$$p(t) = p(0)\, e^{rt},$$

where $p(0)$ is the price at $t = 0$. It then follows that

$$p(0) = p(t)\, e^{-rt},$$

so that $p(0)$ is the *present value* of $p(t)$ at $t = 0$. Since this must hold for any point in time, Hotelling's Rule implies that the discounted resource price is constant along an efficient extraction path.

In *discrete time* notation, we can write this condition as

$$p_0 = \left[\frac{1}{1+\delta}\right]^t p_t, \quad t = 1, 2, ...T,$$

or, equivalently,

$$\frac{p_t}{p_0} = [1+\delta]^t, \quad t = 1, 2, ...T,$$

where δ is the discrete time discount rate and T is an arbitrary final planning period.

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[1] You are encouraged to read more about discounting. See, for example, Conrad p.4 and Perman, *et al.*, p.361 and elsewhere.

Recall that the present value of a stream of payments or profits $v(t)$ over time is given by

$$\int_0^T v(t)\, e^{-rt} dt$$

in continuous time, or

$$\sum_{t=0}^{T} \left[\frac{1}{1+\delta}\right]^t v_t$$

in discrete time. Remember that

$$e^{-r} = \frac{1}{1+\delta} \quad \Leftrightarrow \quad r = \ln(1+\delta),$$

so that the discrete-time rate and the continuous rate are not equivalent.

4 A simple resource allocation problem

Consider a very simple, discrete time, resource allocation problem. A company owns a small non-renewable resource stock of initial size x_0 and intends to extract all of it within just two periods. Assume that the value of the resource when extracted is a function only of the quantity extracted in each period, thus $v_t \equiv v_t(q_t)$. We can write the company's (present value) maximisation problem, choosing q_1 and q_2, as

$$\max_{q_t} \quad \frac{1}{1+\delta} v_1(q_1) + \left[\frac{1}{1+\delta}\right]^2 v_2(q_2) \tag{4}$$

subject to the constraint that

$$q_1 + q_2 = x_0.$$

We can write a *Lagrangian* function for this problem as

$$\mathcal{L} \equiv \frac{1}{1+\delta} v_1(q_1) + \left[\frac{1}{1+\delta}\right]^2 v_2(q_2) + \lambda\left[x_0 - q_1 - q_2\right], \tag{5}$$

where λ is the *Lagrange multiplier* on the stock constraint. For an optimum, we take the derivatives of the Lagrangian with respect to q_1 and q_2 and set them equal to zero. This gives us the two *first order (necessary) conditions*

$$\mathcal{L}_{q_1} = \frac{1}{1+\delta} v_1'(q_1^*) - \lambda = 0 \tag{6}$$

and

$$\mathcal{L}_{q_2} = \left[\frac{1}{1+\delta}\right]^2 v_2'(q_2^*) - \lambda = 0, \tag{7}$$

where $v_t'(q_t)$ is the first derivative of $v_t(q_t)$ with respect to q_t. Solving (6) and (7) for λ and rearranging, we obtain

$$\frac{v_2'(q_2^*)}{v_1'(q_1^*)} = 1+\delta \quad \Leftrightarrow \quad \frac{v_2'(q_2^*) - v_1'(q_1^*)}{v_1'(q_1^*)} = \delta. \tag{8}$$

This is Hotelling's Rule. If we assume that extraction costs are zero, we can define $v_t(q_t) \equiv p_t q_t$, where p_t is the resource price in period t. Then $v_t'(q_t) = p_t$ and (8) becomes

$$\frac{p_2}{p_1} = 1+\delta \quad \Leftrightarrow \quad \frac{p_2 - p_1}{p_1} = \delta$$

as before. In continuous time this is equivalent to

$$\frac{\dot{p}}{p(t)} = r,$$

as we had before.

Alternatively, we could equally well attach a multiplier to a stock constraint at each point in time. Thus, we could write

$$\begin{aligned} \mathcal{L} \equiv & \frac{1}{1+\delta} v_1(q_1) + \left[\frac{1}{1+\delta}\right]^2 v_2(q_2) + \frac{1}{1+\delta}\lambda_1 [x_0 - x_1] \\ & + \left[\frac{1}{1+\delta}\right]^2 \lambda_2 [x_1 - q_1 - x_2] + \left[\frac{1}{1+\delta}\right]^3 \lambda_3 [x_2 - q_2]. \end{aligned} \tag{9}$$

Notice that here we have discounted the multiplier on the stock constraint as well as the value of extraction in each period back to the present. Notice, also, that we have added a final constraint which ensures exhaustion of the resource at $t = 3$ ($x_3 = x_2 - q_2 = 0$). The first order conditions for q_1 and q_2 are now given by

$$\mathcal{L}_{q_1} = \frac{1}{1+\delta} v_1'(q_1^*) - \left[\frac{1}{1+\delta}\right]^2 \lambda_2 = 0 \tag{10}$$

and

$$\mathcal{L}_{q_2} = \left[\frac{1}{1+\delta}\right]^2 v_2'(q_2^*) - \left[\frac{1}{1+\delta}\right]^3 \lambda_3 = 0. \tag{11}$$

If the Lagrangian is maximised by the optimal value of q_1, however, then it should also be maximised by the corresponding value of x_2, so that

$$\mathcal{L}_{x_2} = -\left[\frac{1}{1+\delta}\right]^2 \lambda_2 + \left[\frac{1}{1+\delta}\right]^3 \lambda_3 = 0 \tag{12}$$

should also hold.[2] The condition for x_2^* implies that

$$\lambda_2 = \frac{1}{1+\delta}\lambda_3 \tag{13}$$

and hence, by substitution,

$$v_1'\left(q_1^*\right) = \frac{1}{1+\delta}v_2'\left(q_2^*\right). \tag{14}$$

We have obtained the same result as before, but notice that we have also established that the multiplier on the stock constraint, which is the *shadow price* (marginal value) of the stock at each point in time, is also increasing at the discount rate.

We can easily adapt (9) to set the problem in terms of a *renewable resource* by incorporating a growth function $g_t\left(x_t\right)$ into each of the stock constraints for $t = 1, 2, 3$. Thus,

$$\begin{aligned}\mathcal{L} \equiv{}& \frac{1}{1+\delta}v_1\left(q_1\right) + \left[\frac{1}{1+\delta}\right]^2 v_2\left(q_2\right) + \frac{1}{1+\delta}\lambda_1\left[x_0 + g_0\left(x_0\right) - x_1\right] \\ &+ \left[\frac{1}{1+\delta}\right]^2 \lambda_2\left[x_1 + g_1\left(x_1\right) - q_1 - x_2\right] + \left[\frac{1}{1+\delta}\right]^3 \lambda_3\left[x_2 + g_2\left(x_2\right) - q_2 - x_3\right],\end{aligned} \tag{15}$$

where, notice, we have left open the possibility that x_3 is non-zero! Our first order conditions are now given by

$$\mathcal{L}_{q_1} = \frac{1}{1+\delta}v_1'\left(q_1^*\right) - \left[\frac{1}{1+\delta}\right]^2 \lambda_2 = 0 \tag{16}$$

and

$$\mathcal{L}_{q_2} = \left[\frac{1}{1+\delta}\right]^2 v_2'\left(q_2^*\right) - \left[\frac{1}{1+\delta}\right]^3 \lambda_3 = 0, \tag{17}$$

[2] In a multi-period problem, this would in general hold for all q_t and x_{t+1} since the latter depends on the choice of the former. Here, of course, we have assumed that $q_0 = 0$ so that $x_1 = x_0$ is fixed, as is $x_3 = 0$.

together with

$$\mathcal{L}_{x_2} = -\left[\frac{1}{1+\delta}\right]^2 \lambda_2 + \left[\frac{1}{1+\delta}\right]^3 \lambda_3 \left[1 + g_2'\left(x_2^*\right)\right] = 0. \tag{18}$$

Now we can find

$$\left[\frac{1}{1+\delta}\right]^2 \lambda_2 = \left[\frac{1}{1+\delta}\right]^3 \lambda_3 \left[1 + g_2'\left(x_2^*\right)\right] \tag{19}$$

so that

$$\frac{1}{1+\delta} v_1'\left(q_1\right) = \left[\frac{1}{1+\delta}\right]^2 v_2'\left(q_2\right)\left[1 + g_2'\left(x_2^*\right)\right]$$

$$\Rightarrow \quad v_1'\left(q_1\right)\left[1+\delta\right] = v_2'\left(q_2\right)\left[1 + g_2'\left(x_2^*\right)\right] \tag{20}$$

and therefore

$$\frac{v_2'\left(q_2^*\right)}{v_1'\left(q_1^*\right)} = \frac{1+\delta}{1+g_2'\left(x_2^*\right)} \quad \Leftrightarrow \quad \frac{v_2'\left(q_2^*\right) - v_1'\left(q_1^*\right)}{v_1'\left(q_1^*\right)} = \delta - \frac{v_2'\left(q_2^*\right) g_2'\left(x_2^*\right)}{v_1'\left(q_1^*\right)}. \tag{21}$$

Now our rule for efficient exploitation of the resource takes account of the *growth* of the resource. To simplify this expression a little, notice that

$$\lim_{t_2 \to t_1} \frac{v_2'\left(q_2\right) g_2'\left(x_2\right)}{v_1'\left(q_1\right)} = g'\left(x\right),$$

so that, in continuous time, the rule becomes

$$\frac{dv'\left(q\right)/dt}{v'\left(q\right)} = r - g'\left(x\right), \tag{22}$$

or, if harvesting is costless, so that $v'\left(q\right) = p$,

$$\frac{\dot{p}}{p} = r - g'\left(x\right). \tag{23}$$

We can see more readily that this is just a modified form of Hotelling's Rule, except that, with a growing resource, the resource price does not have to rise at the same rate as the interest rate. Indeed, if $r = g'\left(x\right)$, we do not require the price to increase at all: in effect, the natural growth of the resource is providing the "interest" we require. If $\dot{p} = 0$, (23) can be rearranged to give

$$\frac{p \cdot g'\left(x\right)}{p} = r,$$

which is equivalent to our yield expression (3).

Clearly, the above approach to determining the optimal use of a natural resource over time can be generalised to T periods. We will return to this topic in a later lecture.

5 Further reading

Look at Conrad (1999), pp.1-8, and Hanley, Shogren and White (2007), pp.214-218.