Some Applications of Mathematics in Finance

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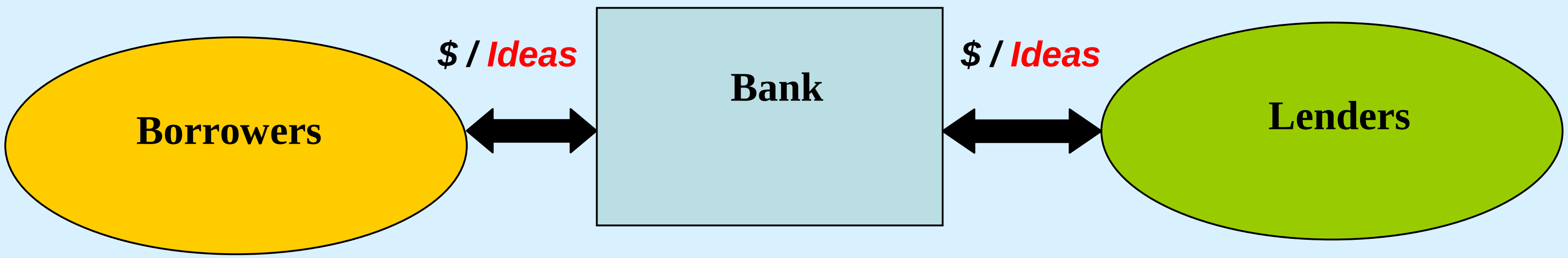
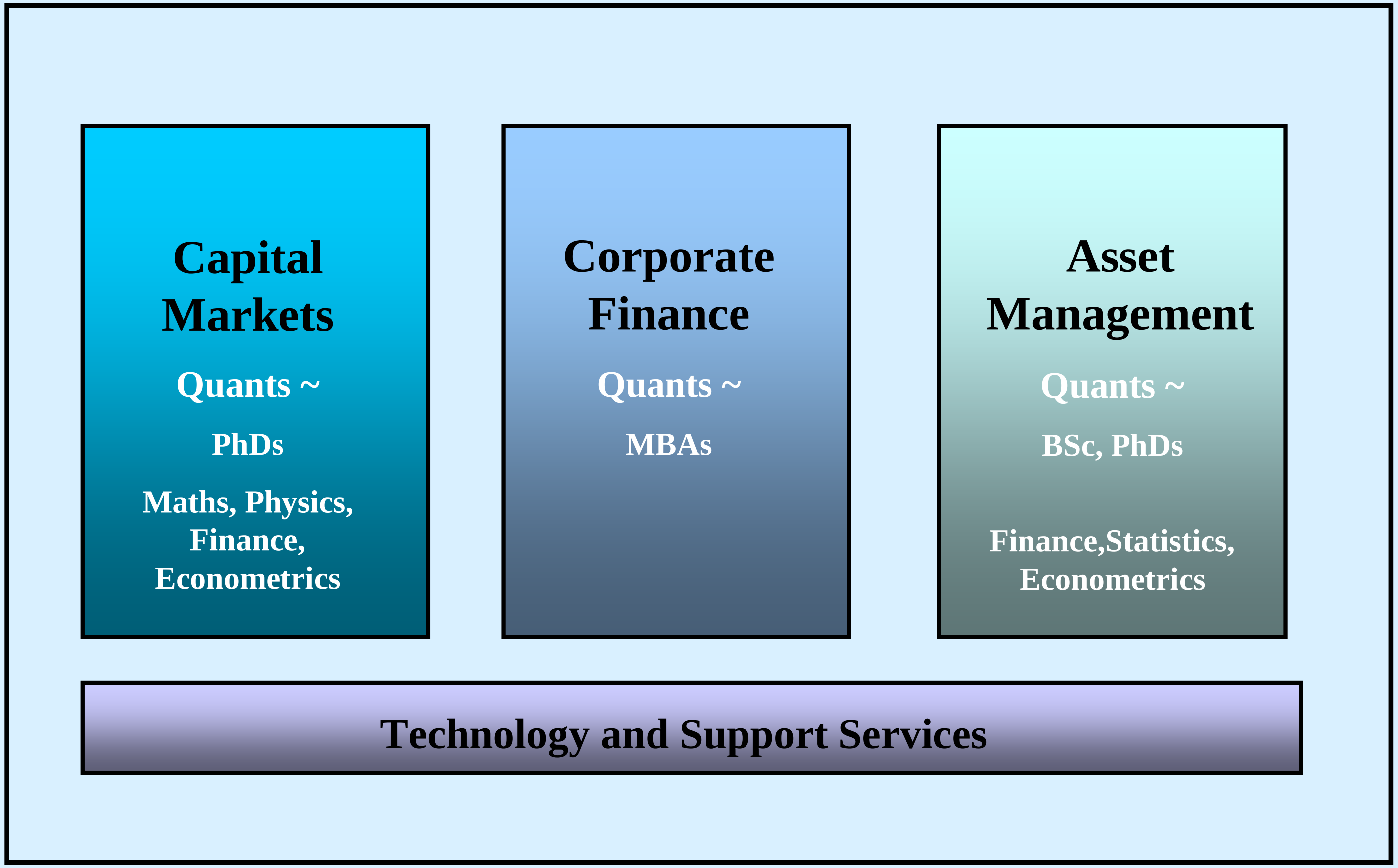
Role of a Bank

Intermediates between institutions that want to borrow capital and institutions that want to lend (invest) capital through

- Provision of high-value ideas and financial structures
- Provision of efficient trade execution



Structure of an (Investment) Bank



Quantitative Modelling within Capital Markets

Rates	Equities	Credit	FX	Commodities
European	European	European,	European,	European,
Bermudian	Bermudian	Bermudian,	Bermudian,	Bermudian,
American	American	American,	American,	American,
Path dependent	Path dependent	Path dependent	Path dependent	Path dependent
Swap Rates	Stock Prices	Credit Spreads	FX Rates	Commodity Prices
Bond Yields	Stock Indices	Corporate Defaults	FX Volatility	Commodity Volatility
Mortgages	Equity Volatility	Default Correlations	FX Correlation	
Interest Rate Volatility	Equity Correlation			
Interest Rate Correlation				

Hybrids

Structured on combination of Rates, Equities, Credit, FX and Commodities

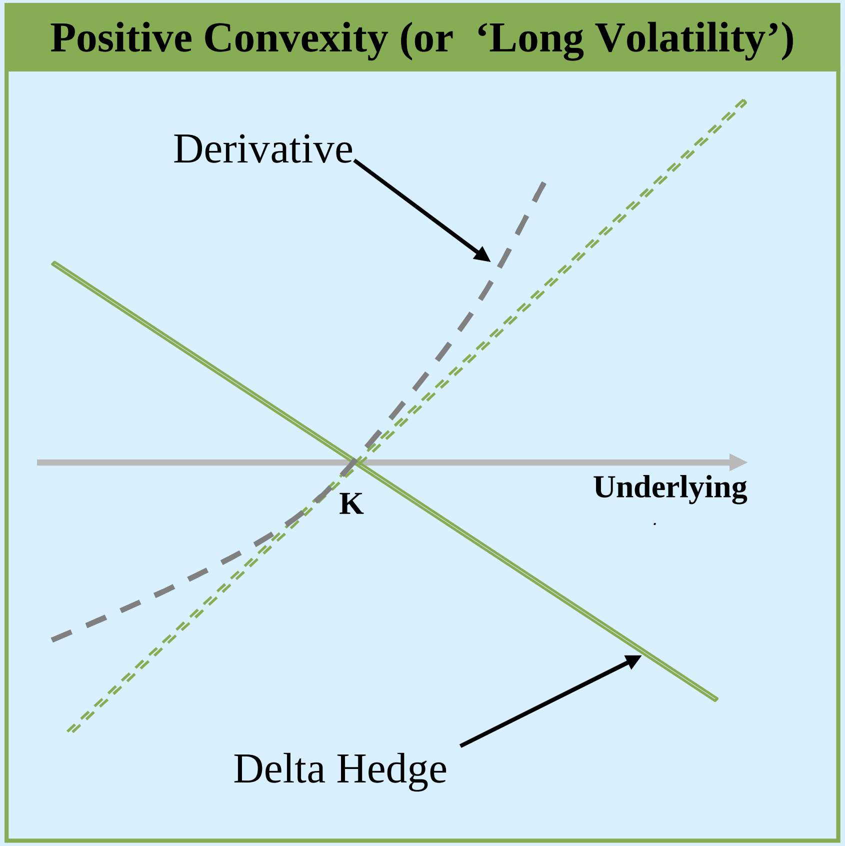
Why are Quants Necessary?

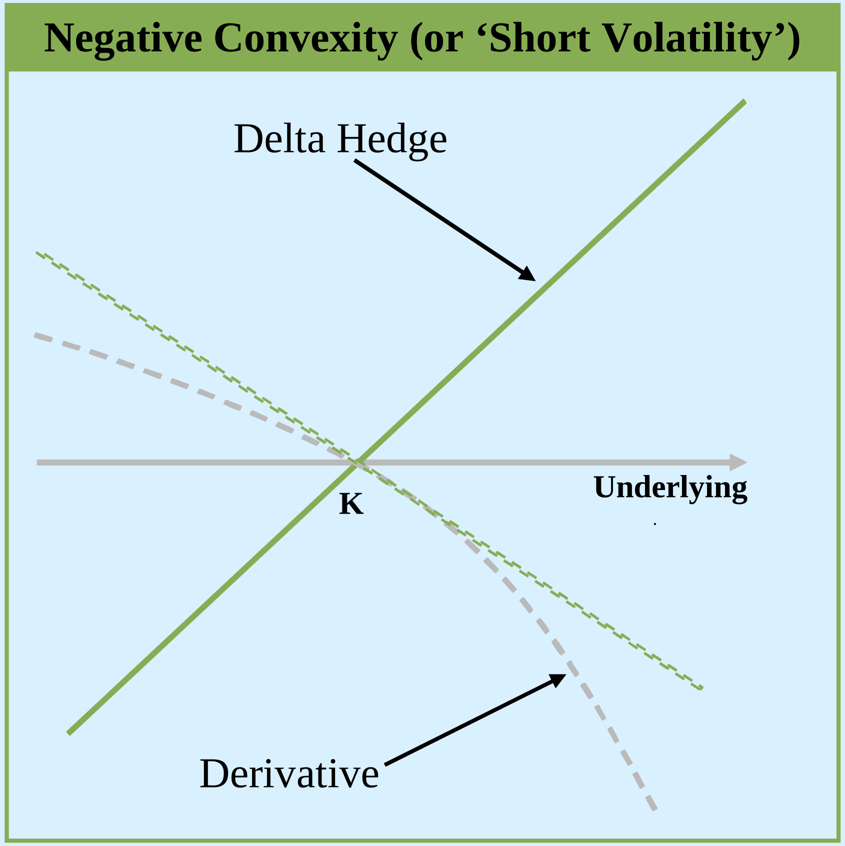
1. **Determining value of derivative ‘convexity’**
   - Derivative values are non-linear (with respect to underlying asset)
   - Hedging instruments have linear behaviour
   - Dynamic (‘delta-hedging’) creates a ‘convexity mis-match’
     - (non-linear derivative vs linear hedge)

2. **Tremendous growth in Structured Notes market since 1993**
   - Complex optionality embedded within structured notes

3. **Electronic Trading**
   - Investors can now buy $100,000,000 of Government bonds in <1 second by ‘clicking’ on various electronic trading portals (e.g. Tradeweb)
   - Need maintain very tight, accurate and tradable prices that update every 100 milliseconds

Positive and Negative Convexity





Learning the different terminologies is important…

- Long Convexity
- Long Volatility
- Long Gamma
- Long Vega
- Long Optionality
- Long Curvature

- **All mean the same thing!**

Assumptions behind Options Valuation Models

1. Asset prices are log-normally distributed
2. Continuous trading in all quantities
3. No bid-offer spreads or other trading costs (commissions, taxes, etc)
4. Constant volatility
5. Constant interest rates

Mathematical Model

- Asset price process modelled via lognormal stochastic differential equation

$$dX_t = \mu X_t dt + \sigma X_t dB_t$$

- Value of call option is the solution of the 'terminal value problem'

$$C_t = rC - (\mu - \lambda\sigma) X_t C_X - \frac{1}{2}\sigma^2 X_t^2 C_{XX}$$

$$C(X_T, T) = (X_T - K)_+$$

Solution (Black's Model)

$$C(X_t, t, T) = E^Q[e^{-\int_t^T r du}(X_T - K)_+ / \Im_t]$$

$$dX_t = (\mu - \lambda\sigma)X_t dt + \sigma X_t dB_t^Q$$

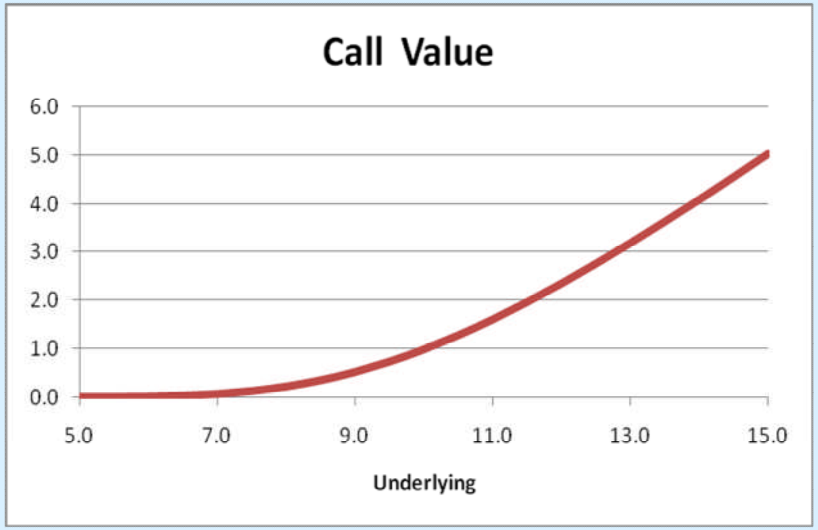
$$dB_t^Q = dB_t + \lambda dt$$

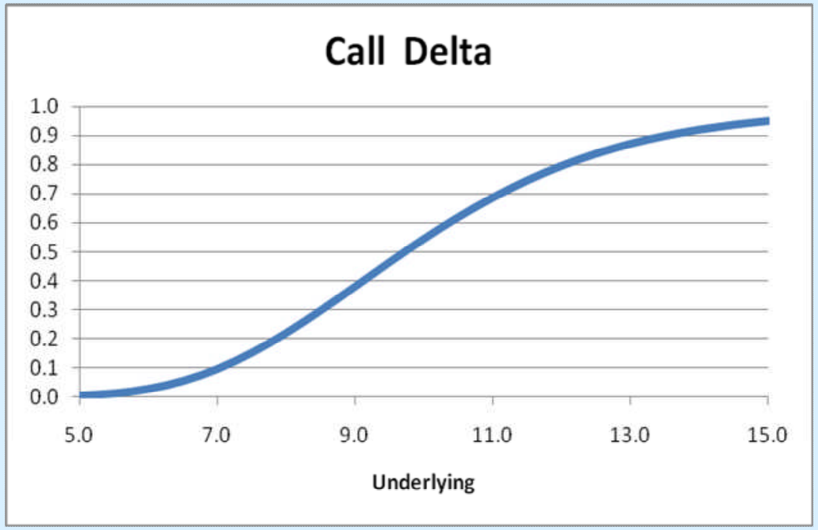
$$C(X_t, t) = X_t N(d_1) - Ke^{-r(T-t)} N(d_2)$$

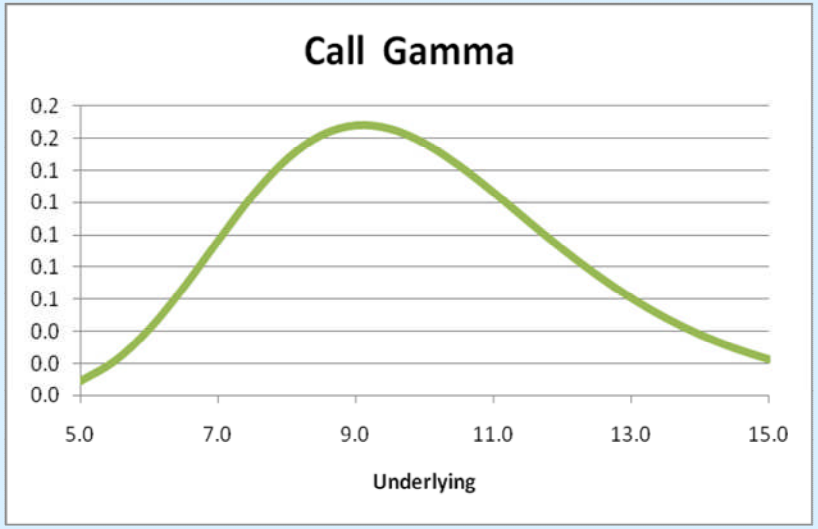
$$d_1 = [\ln(X_t / K) + (r \; + \frac{1}{2}\sigma^2(T-t)] / (\sigma\sqrt{T-t})$$

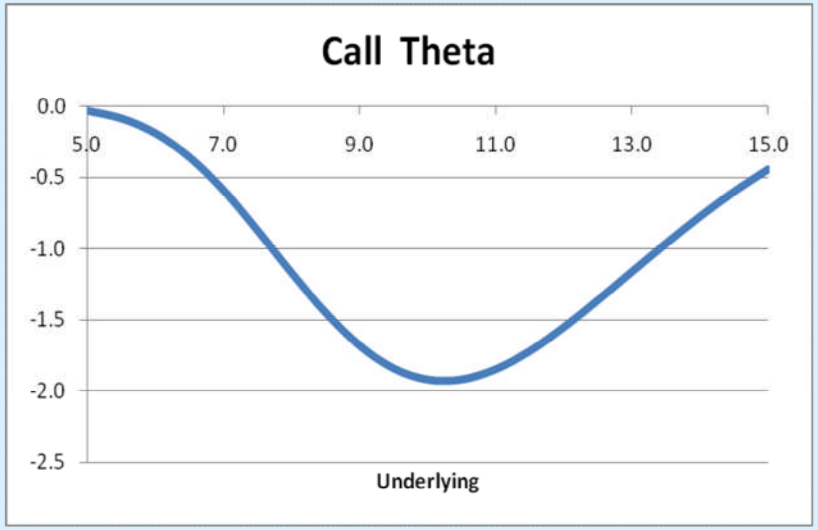
$$d_2 = d_1 - \sigma\sqrt{T-t}$$

Value, Delta, Gamma, Theta

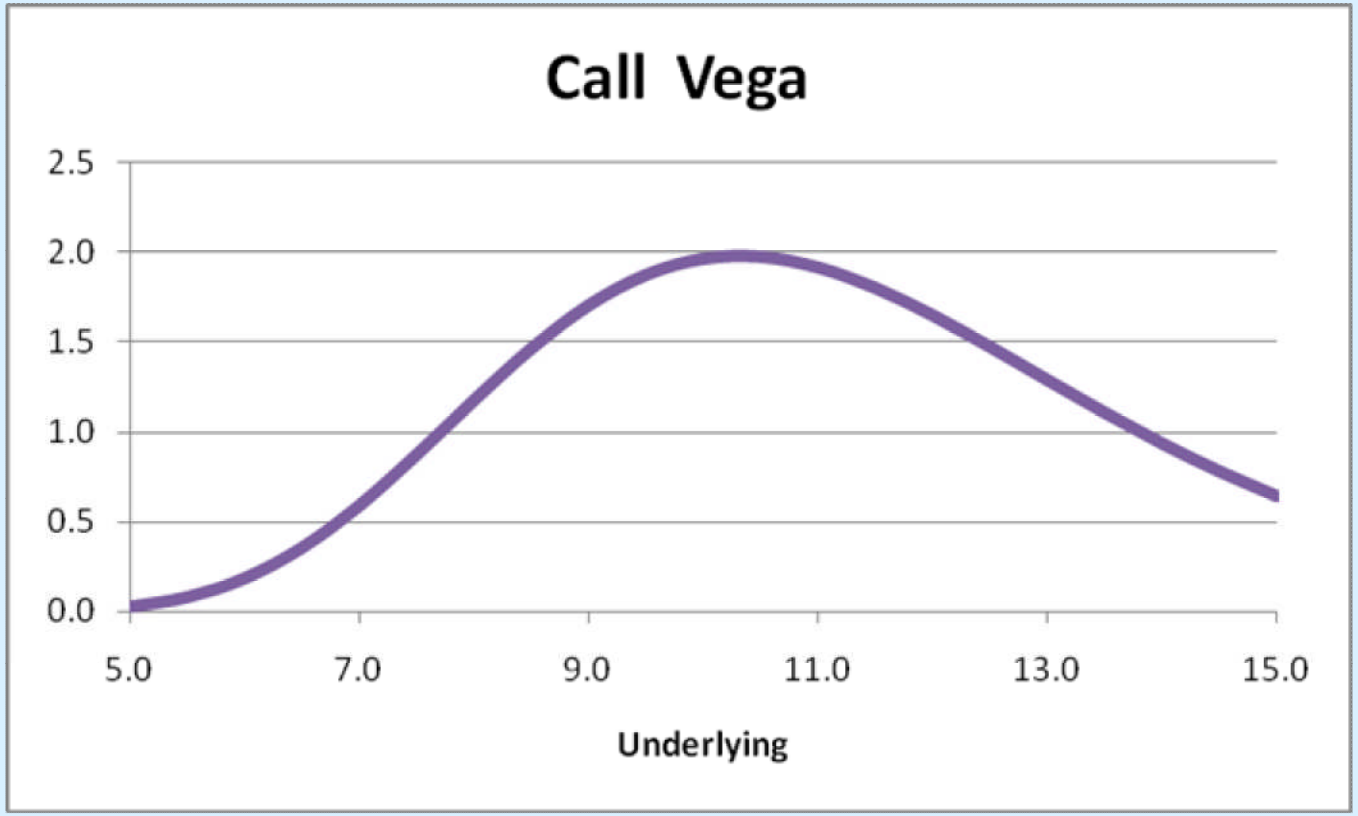




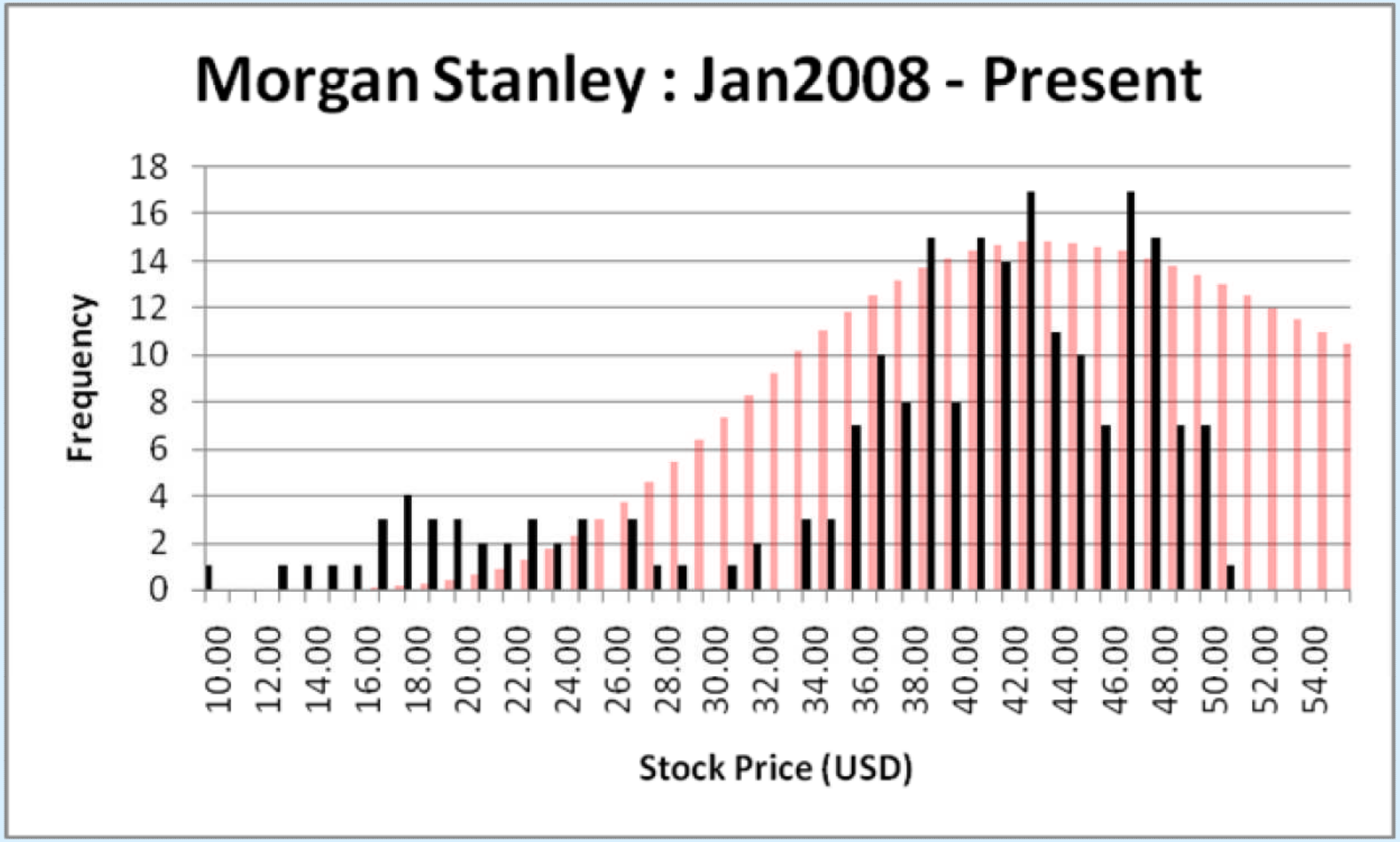




Vega

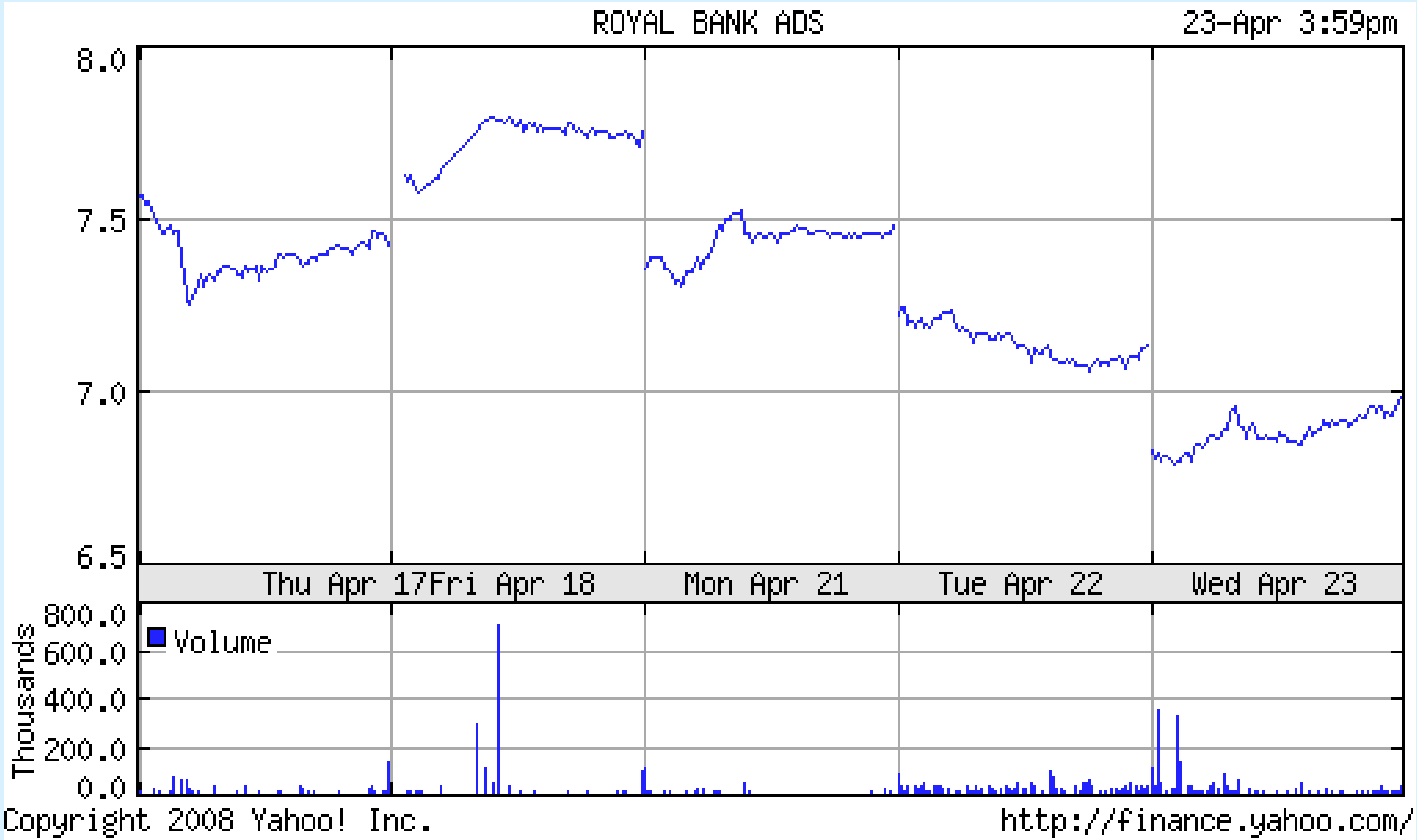


Lognormal Distribution typically under estimates tails

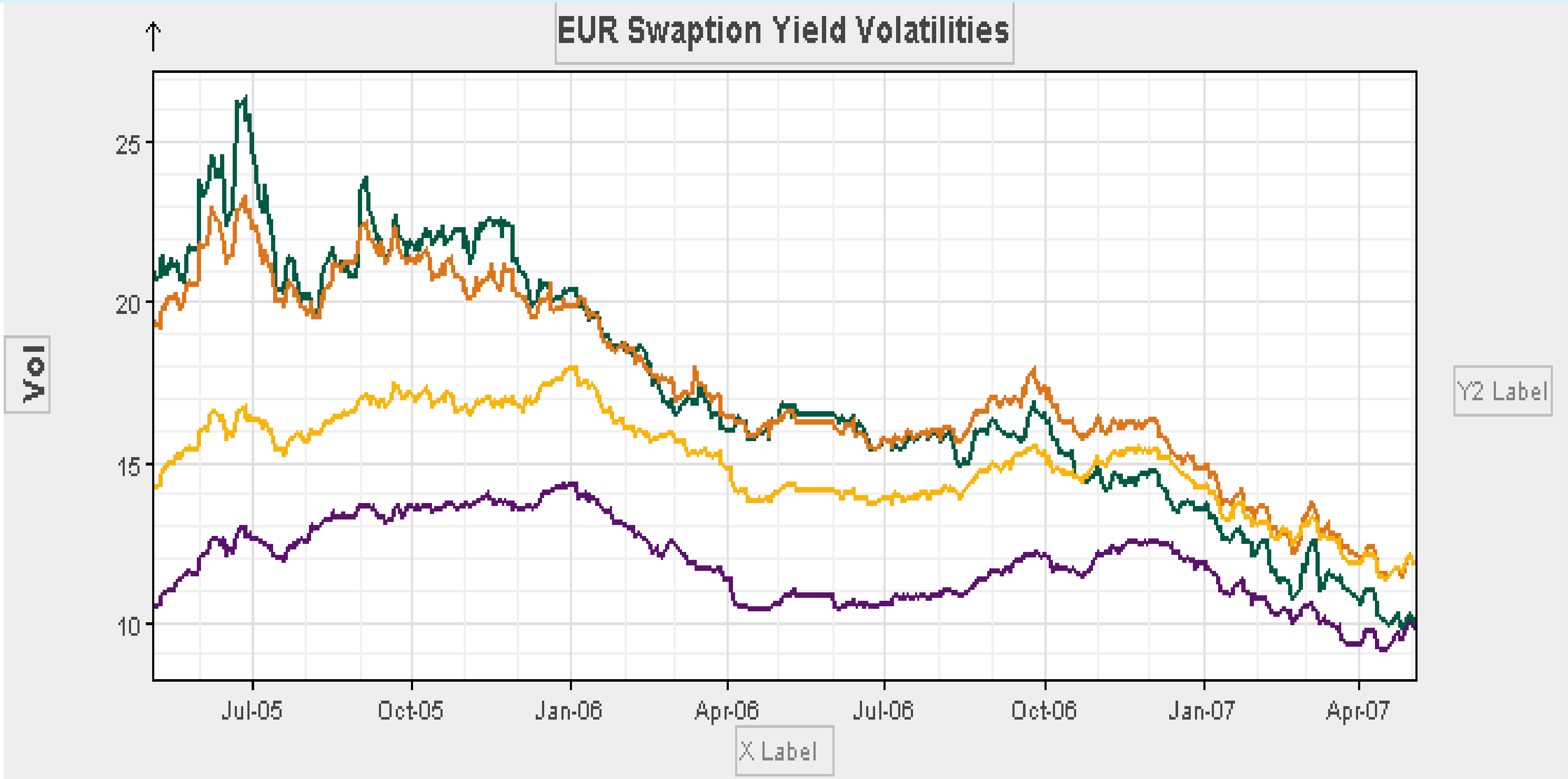


Stock Prices are Not Continuous…

(RBS 17th April - 23rd April 2008)



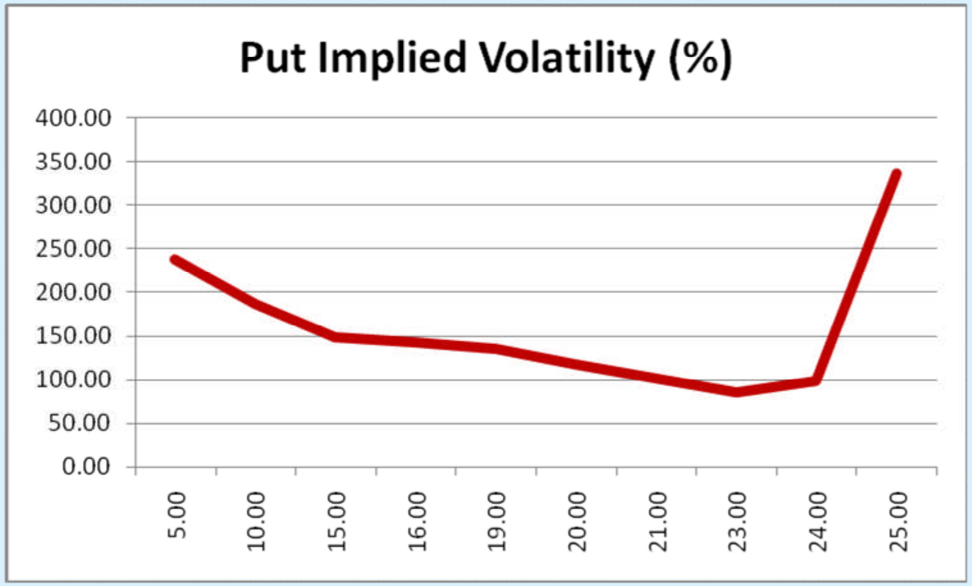
Implied volatilities are not constant…

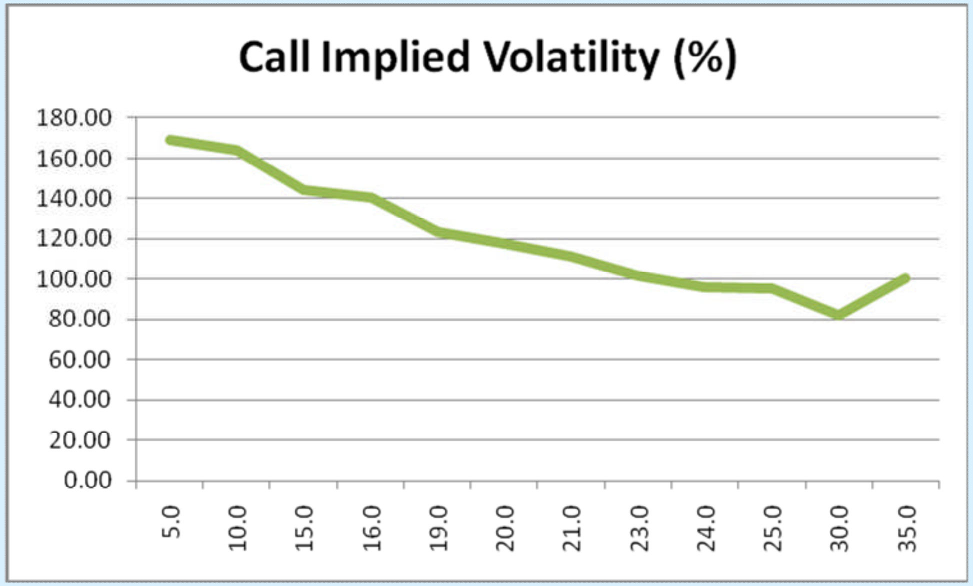


Expressions | Statistics | Options | Annotator

Key	Axis	Name	Last	Minimum		Maximum		Mean	SD	SD Change
	Left	1Y Rate 1Y Forward	10.100	9.800	26-Apr-2007	26.400	27-Jun-2005	17.245	3.893	0.308
	Left	2Y Rate 2Y Forward	11.900	11.400	18-Apr-2007	23.300	27-Jun-2005	17.412	2.969	0.223
	Left	5Y Rate 5Y Forward	11.900	11.300	16-Apr-2007	17.900	30-Dec-2005	15.017	1.532	0.143
	Left	20Y Rate 10Y Forward	9.800	9.100	13-Apr-2007	14.400	28-Dec-2005	11.849	1.299	0.111

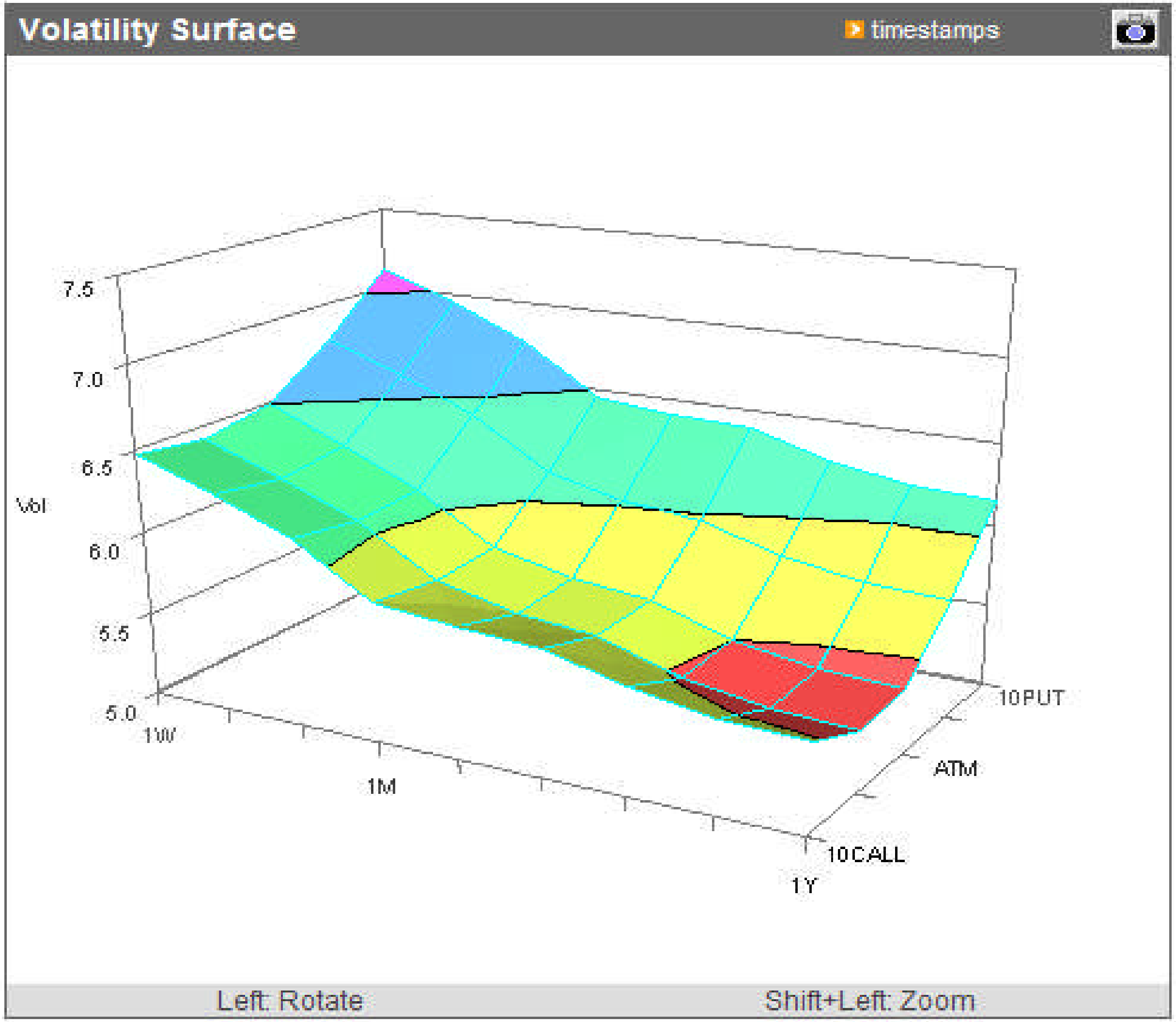
Volatility Skews





Implied volatilities constitute a 3-dimensional surface…

Foreign Exchange Volatility and Correlation (Keyword: fxvol)



Volatility Tools and Models

Volatility	FX Spirit
Scorecard	Time Series Plotter
Vol Cones	Event Calendar
Vol Surface	CurveLab
EM FX Vol	Market Monitor

Currencies

GBP/CHF   Bid   Mid   Ask

Live

Volatilities

Smile   RR/STR

Maturity	10 GBP put	25 GBP put	ATM	25 GBP call	10 GBP call
1W	7.12	6.77	6.50	6.42	6.48
2W	6.95	6.61	6.30	6.26	6.32
3W	6.76	6.42	6.10	6.07	6.13
1M	6.46	6.12	5.80	5.78	5.83
2M	6.42	6.02	5.70	5.68	5.78
3M	6.39	5.98	5.65	5.62	5.76
6M	6.27	5.82	5.50	5.48	5.64
9M	6.18	5.75	5.42	5.40	5.56
1Y	6.17	5.72	5.40	5.38	5.54

Data for 25-Apr-2007

There are many measures of volatility…

Currency: EUR London | Item: Yield Volatility (implied lognormal) | Date: 03May2007 | Customize

- Yield Volatility (implied lognormal)
- Basis-Point Volatility (implied)
- Forward Swap Yield
- 1M (20 days) Realised Log Vol
- 2M (40 days) Realised Log Vol
- 3M (60 days) Realised Log Vol
- 6M (120 days) Realised Log Vol
- 1Y (250 days) Realised Log Vol
- 1M (20 days) Realised BP Vol
- 2M (40 days) Realised BP Vol
- 3M (60 days) Realised BP Vol
- 6M (120 days) Realised BP Vol
- 1Y (250 days) Realised BP Vol

Option Maturity	1Y	2Y	3Y	4Y	5Y	6Y	7Y	8Y	9Y	10Y	15Y	20Y	25Y	30Y
Yield Volatility (implied lognormal)														3D Graph
3M	6.60	9.10	10.20	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	11.00	10.90	10.50	10.20	10.20	10.10
6M	7.90	9.60	10.40	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	11.10	11.00	10.70	10.40	10.20	10.10
1Y	10.10	10.80	11.30	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	11.30	11.20	10.80	10.70	10.50	10.30
2Y	11.50	11.90	11.90	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	11.60	11.50	11.10	10.90	10.60	10.50
3Y	12.00	12.30	12.30	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	11.60	11.50	11.10	10.90	10.70	10.40
4Y	12.20	12.30	12.20	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	11.60	11.50	11.10	10.80	10.60	10.40
5Y	12.40	12.40	12.30	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	11.50	11.30	11.00	10.70	10.60	10.40
7Y	12.10	12.00	11.80	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	11.10	11.00	10.60	10.30	10.30	10.00
10Y	11.30	11.20	11.10	11.00	[illegible]	[illegible]	[illegible]	10.60	10.60	10.50	10.10	9.80	9.70	9.50
15Y	10.30	10.30	10.20	10.10	10.10	10.00	10.00	9.90	9.90	9.90	9.50	9.20	9.00	8.80
20Y	9.80	9.90	9.70	9.60	9.60	9.60	9.60	9.60	9.50	9.50	9.50	8.60	8.50	8.40
Cap/Floor Volatilities														
-	6.00	8.54	10.09	10.80	11.29		11.78			11.90	11.64			

Thu 03 May 2007

Break-even Volatility

Buy derivative at V and delta hedge with underlying asset X

=> ‘Long Convexity, short Theta’

Change in hedged portfolio over small time interval is

$$\delta P_t = \delta V(X_t, \sigma, t) - V_X \delta X_t + r(V_X X_t - V)\delta t$$
$$= (V_t + rXV_X - rV)\delta t + \frac{1}{2} V_{XX} \delta X^2 + \ldots$$

Hedged portfolio makes money if underlying asset moves more than

$$\left(\frac{\delta X}{X}\right)^2 > \frac{-(V_t + rXV_X - rV)\delta t}{\frac{1}{2} X^2 V_{XX}}$$

This is known as the ‘breakeven volatility’ (~ Theta / Gamma)

- Traders compute break-even volatility before they put on a trade.

Option value as a Break-even price...

- Buy call option at price V and delta hedge with underlying asset X.

- Change in hedged portfolio over small time interval is

$$\delta P_t = \delta V(X_t, \sigma, t) - V_X \delta X_t + r(V_X X_t - V)\delta t + V_\sigma \delta\sigma + V_r \delta r + \ldots$$
$$= (V_t + rXV_X - rV)\delta t + \frac{1}{2} V_{XX} \delta X^2 + \ldots$$

- Zero profit or loss on hedged portfolio over small time interval if option value satisfies the equation

$$(V_t + rXV_X - rV)\delta t + \frac{1}{2} V_{XX} \delta X^2 = 0$$

$$V_t + rXV_X - rV + \frac{1}{2}\sigma^2 X^2 V_{XX}{}^2 = 0$$

Break-even Volatility

Example – Long Call Option

Spot = 18.00 USD

Strike = 18.00 USD

T = 0.25 years

Log Volatility = 50.00%

Daily ‘break-even’ stock move must be 0.44 USD (i.e. 44cents)

=> Annualised Stock Volatility ~ 47.3%

If stock moves less that this –

Theta ‘decay’ and option ‘carry’ will swamp what you’ll make every time you re-balance your delta hedge!

Profit and Loss on a Delta-Hedge

Buy derivative at V and delta hedge with underlying asset X

=> ‘Long Convexity, short Theta’

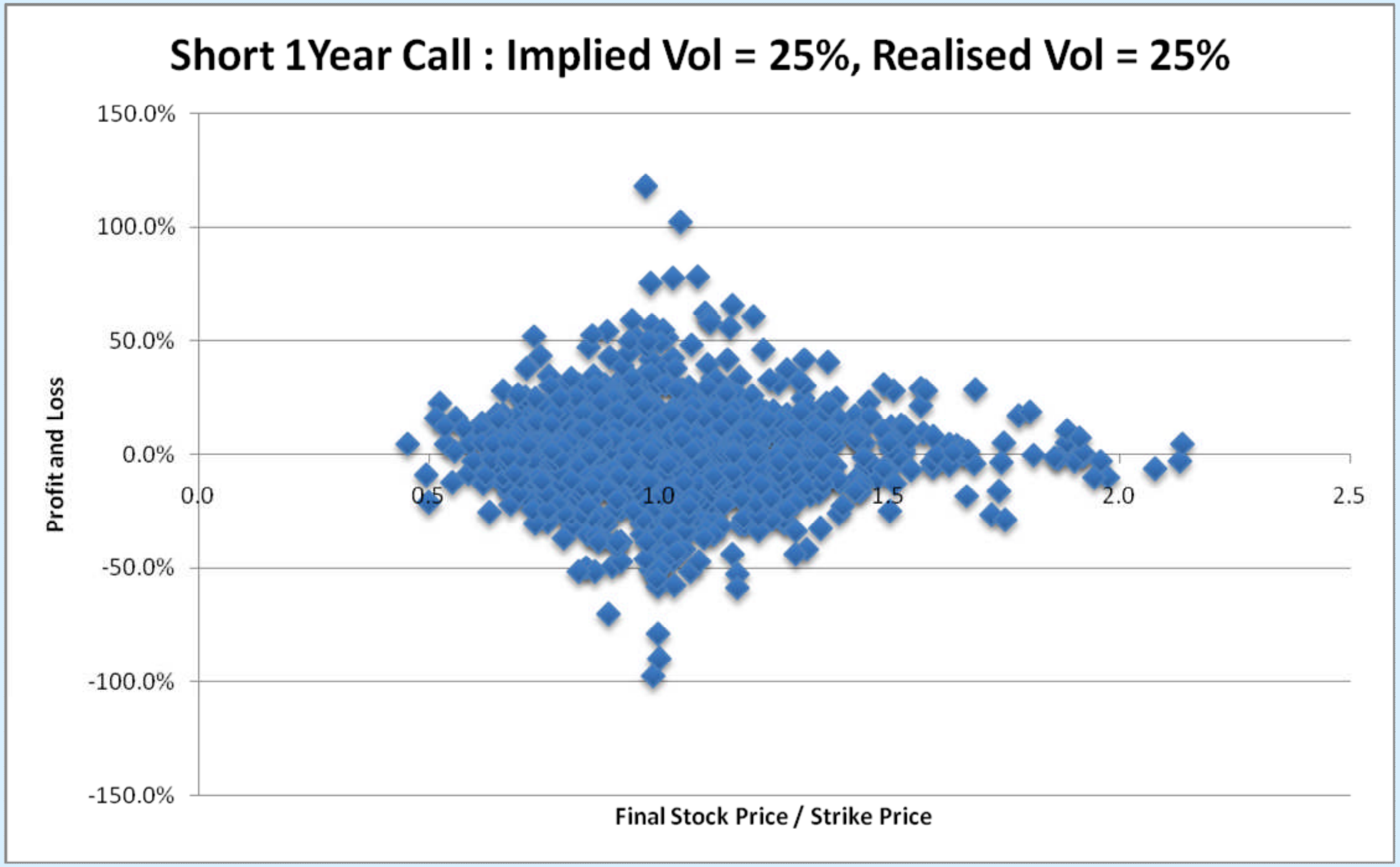
Change in hedged portfolio over small time interval is

$$
\begin{aligned}
\delta P_t &= \delta V(X_t, \sigma, t) - V_X \delta X_t + r(V_X X_t - V)\delta t \\
&= V_t \delta t + V_X \delta X_t + \frac{1}{2} V_{XX} \delta X^2 + \ldots - V_X \delta X_t + r(V_X X_t - V)\delta t \\
&= \frac{1}{2} X^2 V_{XX} \left\{ \left(\frac{\delta X}{X} \right)^2 - \sigma_I^2 \right\} \delta t + \ldots
\end{aligned}
$$

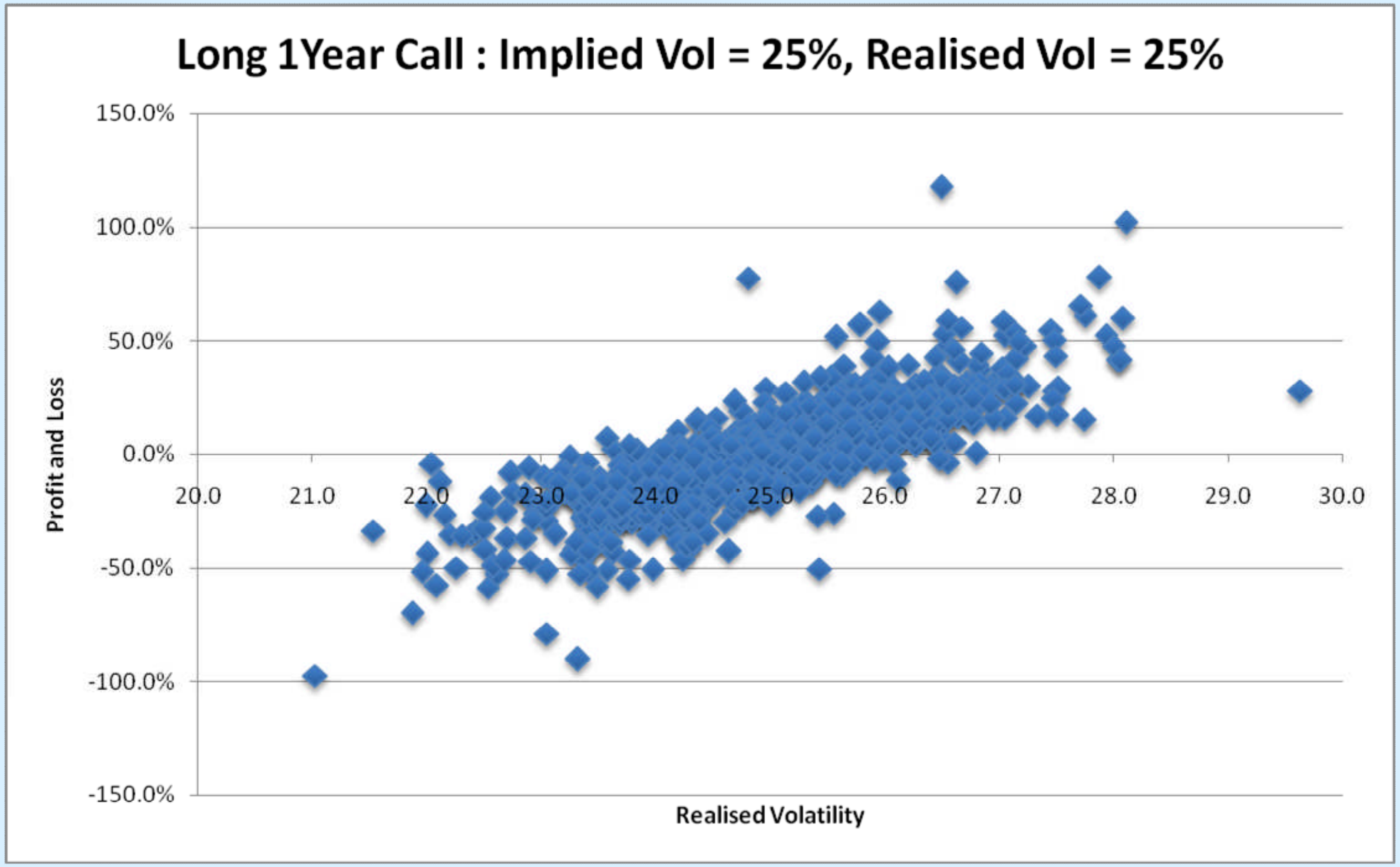
•PnL on a delta-hedge is **gamma weighted difference between the realised volatility and the implied volatility**

•True PnL must incorporate bid-offer costs

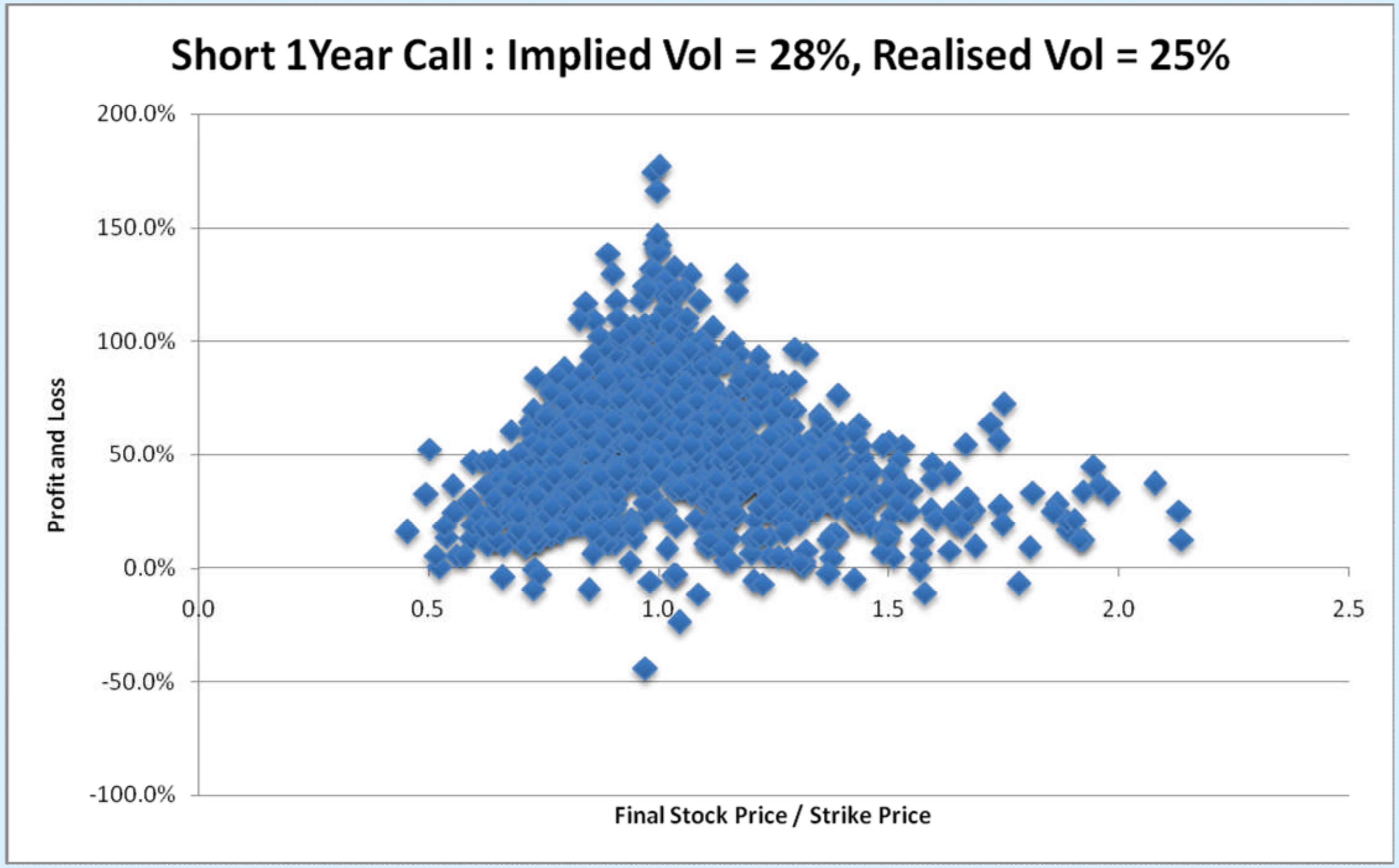
Delta Hedging - 1



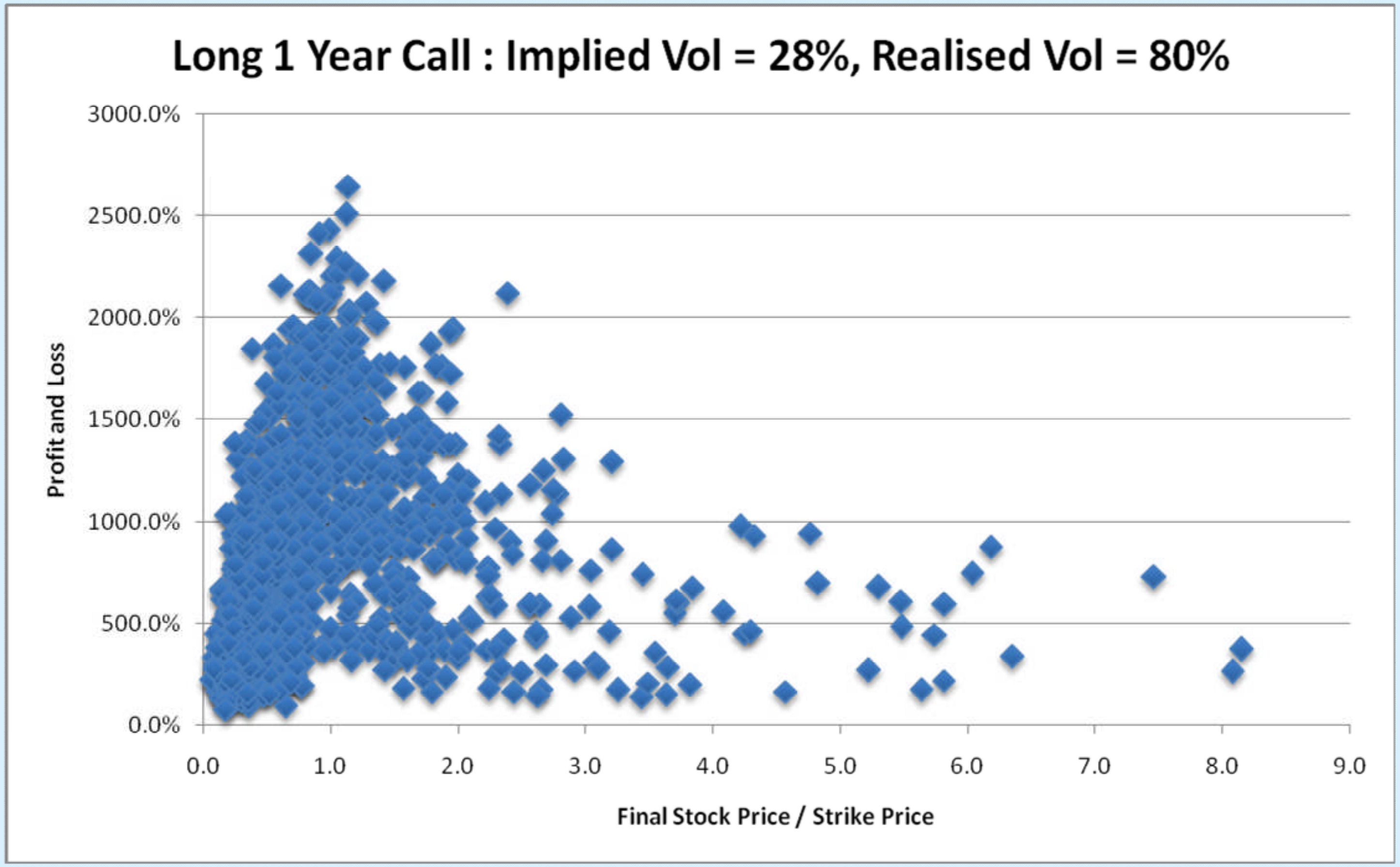
Realised Volatility > Implied Volatility



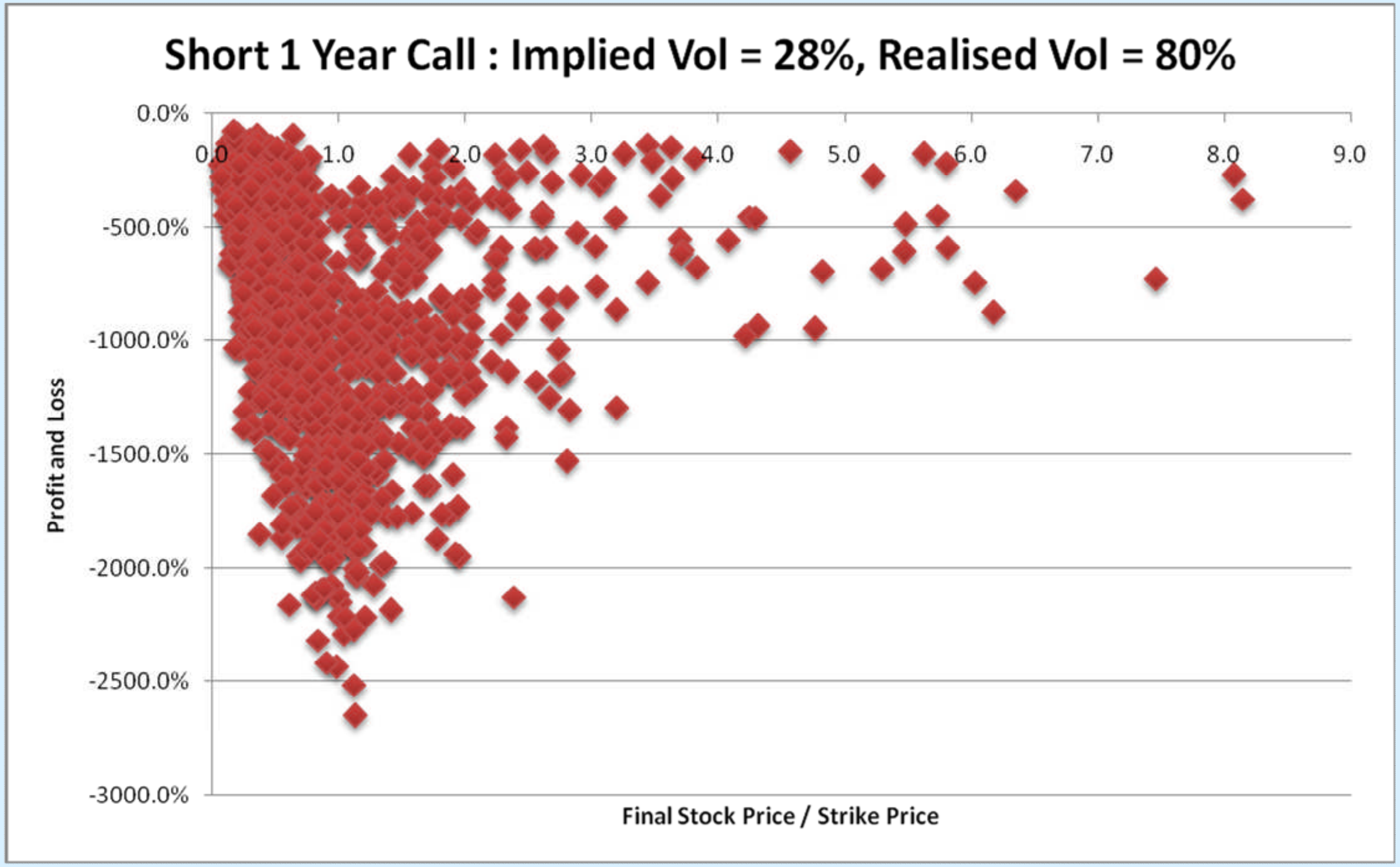
Delta Hedging - 2



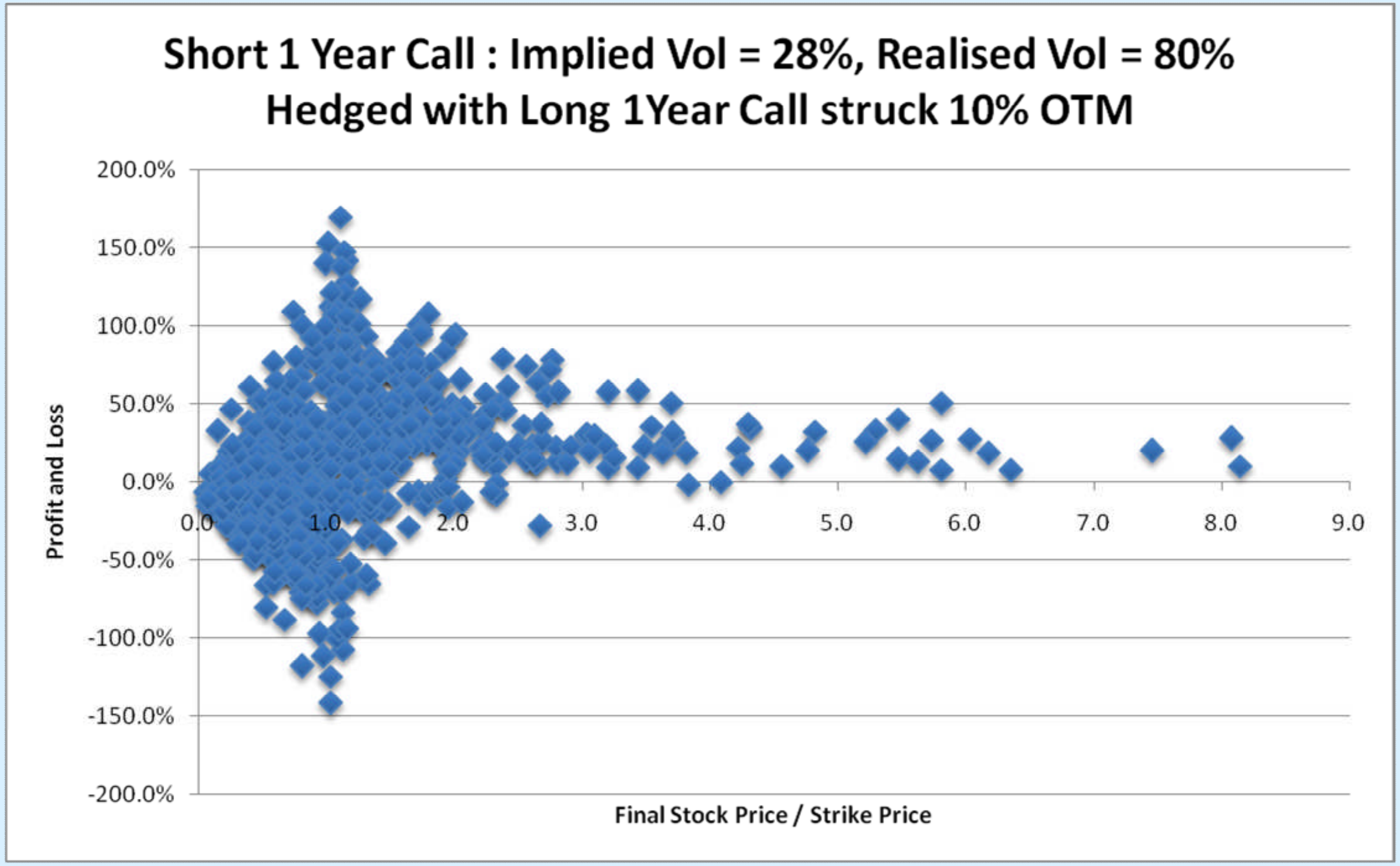
Delta Hedging - 3



Delta Hedging - 4



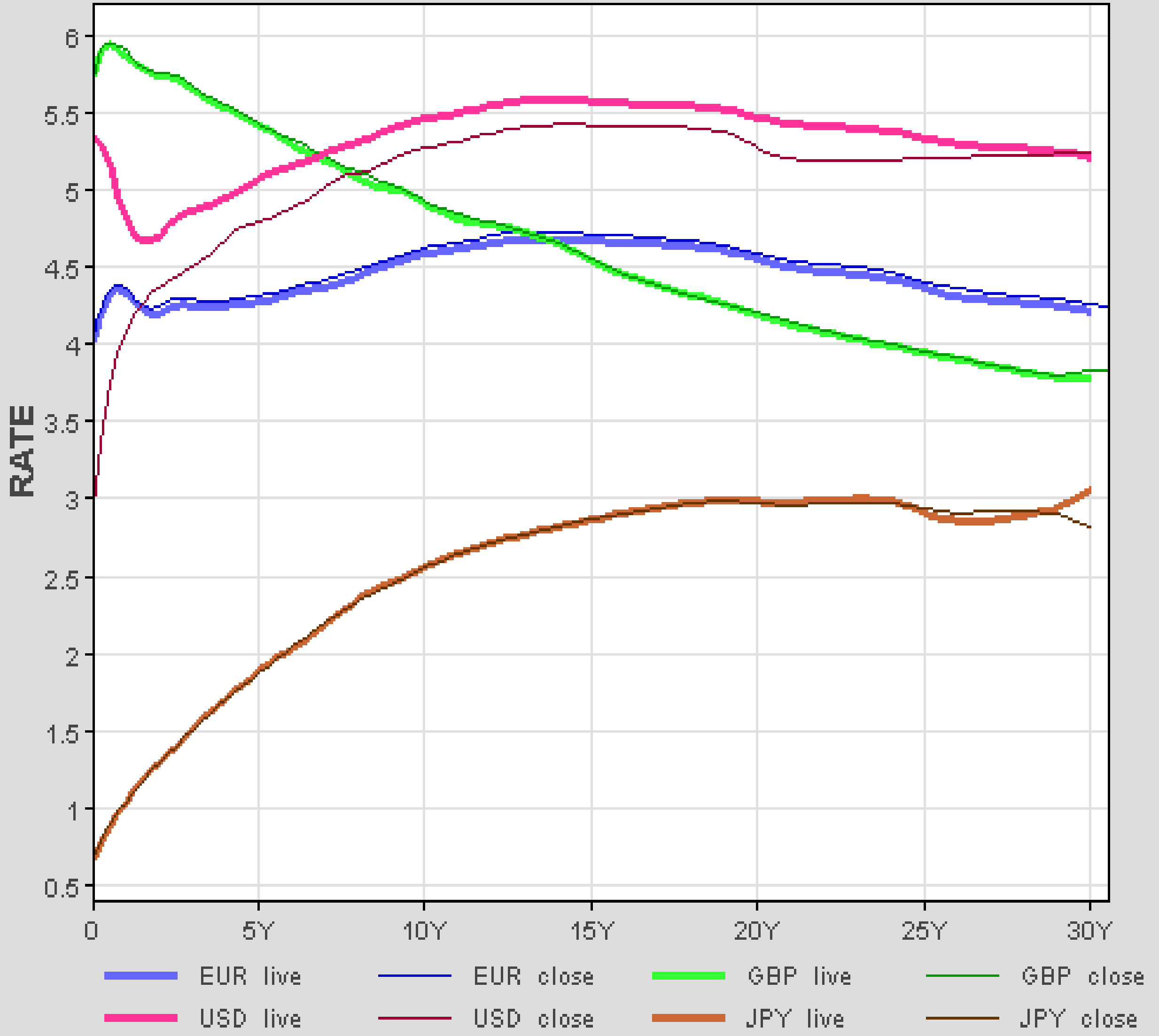
Delta Hedging - 5



Forward Rates (3 Month LIBOR)

Live Swap Curves

Forward Curves and Break Even Swap Rates



Term	EUR	USD	GBP	JPY
1Y	4.372	5.319	5.917	0.832
2Y	4.383	5.074	5.887	0.995
3Y	4.377	5.012	5.850	1.126
4Y	4.376	5.007	5.802	1.244
5Y	4.381	5.025	5.755	1.352
6Y	4.387	5.053	5.705	1.452
7Y	4.399	5.083	5.655	1.541
8Y	4.413	5.113	5.607	1.628
9Y	4.433	5.145	5.560	1.712
10Y	4.454	5.176	5.517	1.787
12Y	4.495	5.231	5.435	1.919
15Y	4.545	5.297	5.332	2.077
20Y	4.587	5.356	5.175	2.264
25Y	4.590	5.375	5.045	2.380
30Y	4.575	5.377	4.940	2.445

Forward: 3 month

- EUR, USD, GBP, JPY
- SEK, DKK, CHF, NOK

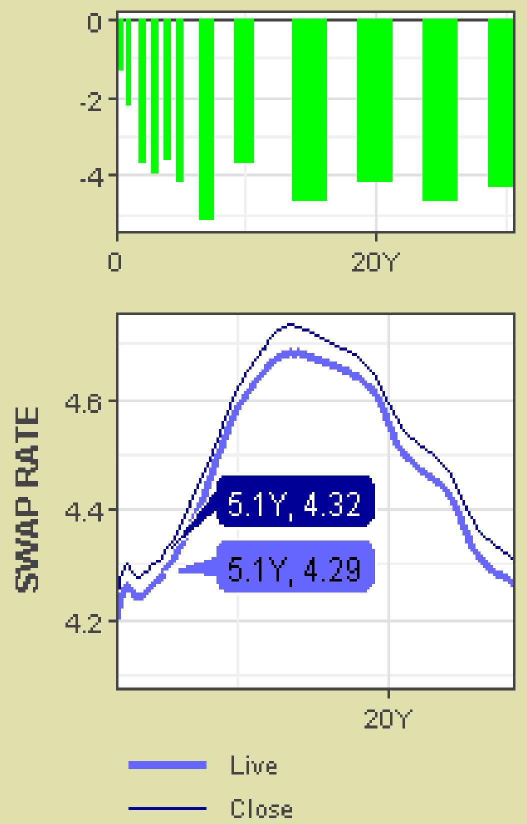
Interest Rate Derivatives

Levels are indicative. Currency: ◉ EUR ○ GBP Customize

Spreads (maturity adjusted swap - bond yield)					Spread Change			1Y Historic Vol (bp)		
Term	Issue	Bond	Swap	Sprd	1D	1M	3M	Bond	Swap	Sprd
2Y	3.750% Schatz 13-Mar-2009	4.147	4.383	23.3	0.8	0	1	44.0	45.8	16.6
5Y	4.000% Bobls 13-Apr-2012	4.163	4.383	21.9	-0.8	1	2	52.7	51.3	11.5
10Y	3.750% Bund 04-Jan-2017	4.192	4.457	25.7	-1.8	1	3	50.9	49.4	10.9
30Y	4.000% Bund 04-Jan-2037	4.367	4.578	21.3	-0.8	1	0	48.8	48.9	10.8

- Time Series Plotter
- CurveLab
- Event Calendar
- Prop Shop
- PS Trade Tracker
- EU Rates Strategy

EUR 3M Swap Forward



Relative Swap Rates, 1Y stats

Term	Spread	Mean	SD	High	Low
2Y-5Y	0.0	8	12	35	-5
2Y-10Y	7.4	23	20	69	2
5Y-10Y	7.4	15	8	34	4
10Y-30Y	12.1	19	8	35	8

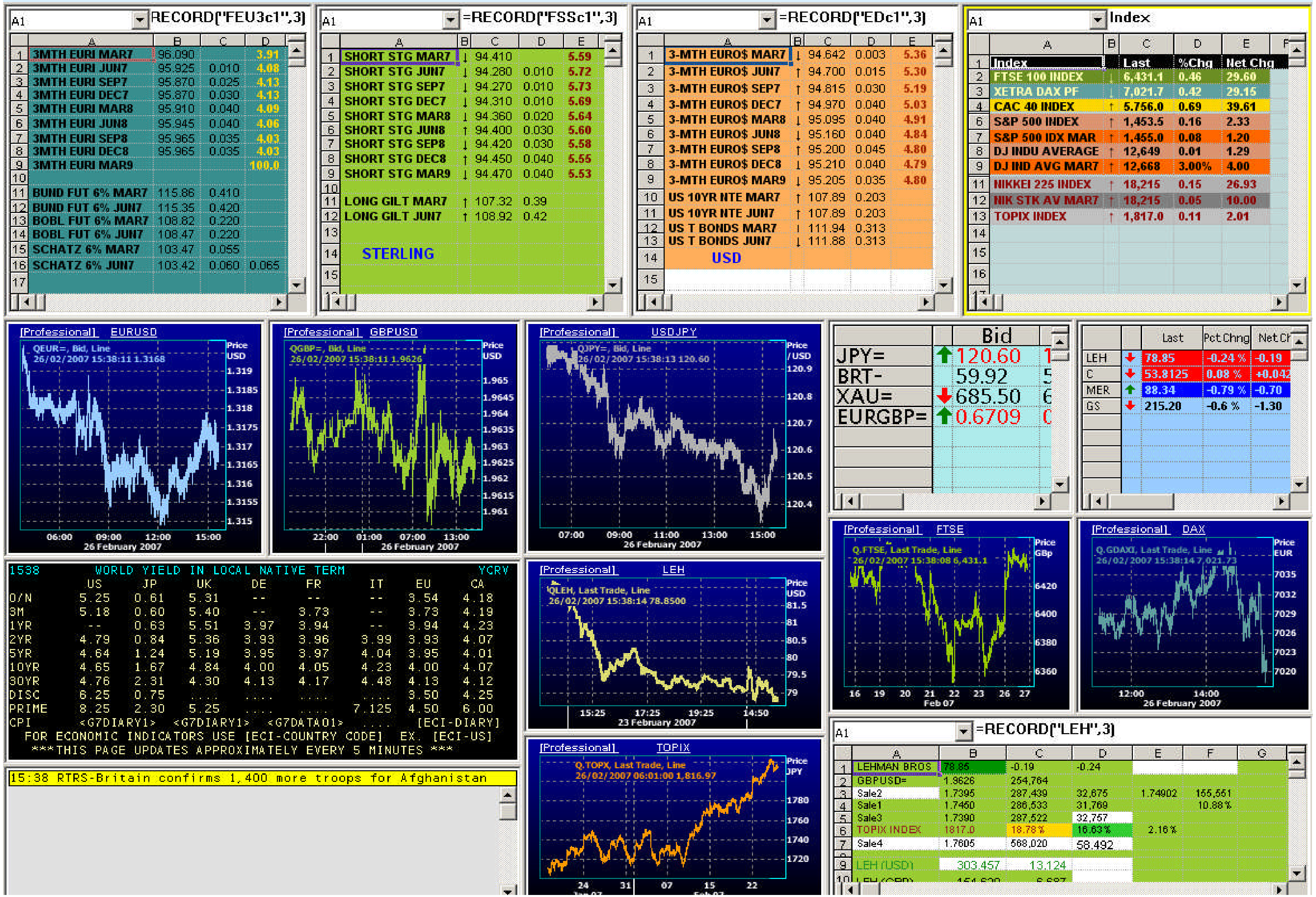
LIBOR Caps and Floors

Term	Implied Vol	1D	1M	3M
2Y	8.5	0.0	-0.8	-2.0
5Y	11.3	0.0	-0.6	-1.8
10Y	11.9	0.0	-0.3	-1.5

European ATM Swaptions

Swap Term	Option Maturity	Implied Vol	1D Change	1M Change	3M Change	Forward Swap	Basis Pt Vol
2							
2Y	1Y	10.8	0.0	-0.8	-2.4	4.383	47.7
2Y	5Y	12.4	0.1	0.1	-1.1	4.453	55.7
2Y	10Y	11.2	0.1	0.2	-1.1	4.768	53.8
5							
5Y	1Y	11.8	0.0	-0.3	-1.6	4.394	52.3
5Y	5Y	11.9	0.1	0.0	-1.2	4.548	54.6
5Y	10Y	10.9	0.1	0.3	-0.8	4.805	52.9
10							
10Y	1Y	11.2	0.0	-0.1	-1.6	4.491	50.7
10Y	5Y	11.3	0.0	0.1	-1.2	4.662	53.1
10Y	10Y	10.5	0.0	0.3	-0.9	4.800	50.9

Typical MarketData Sheet (24Febuary 2007)

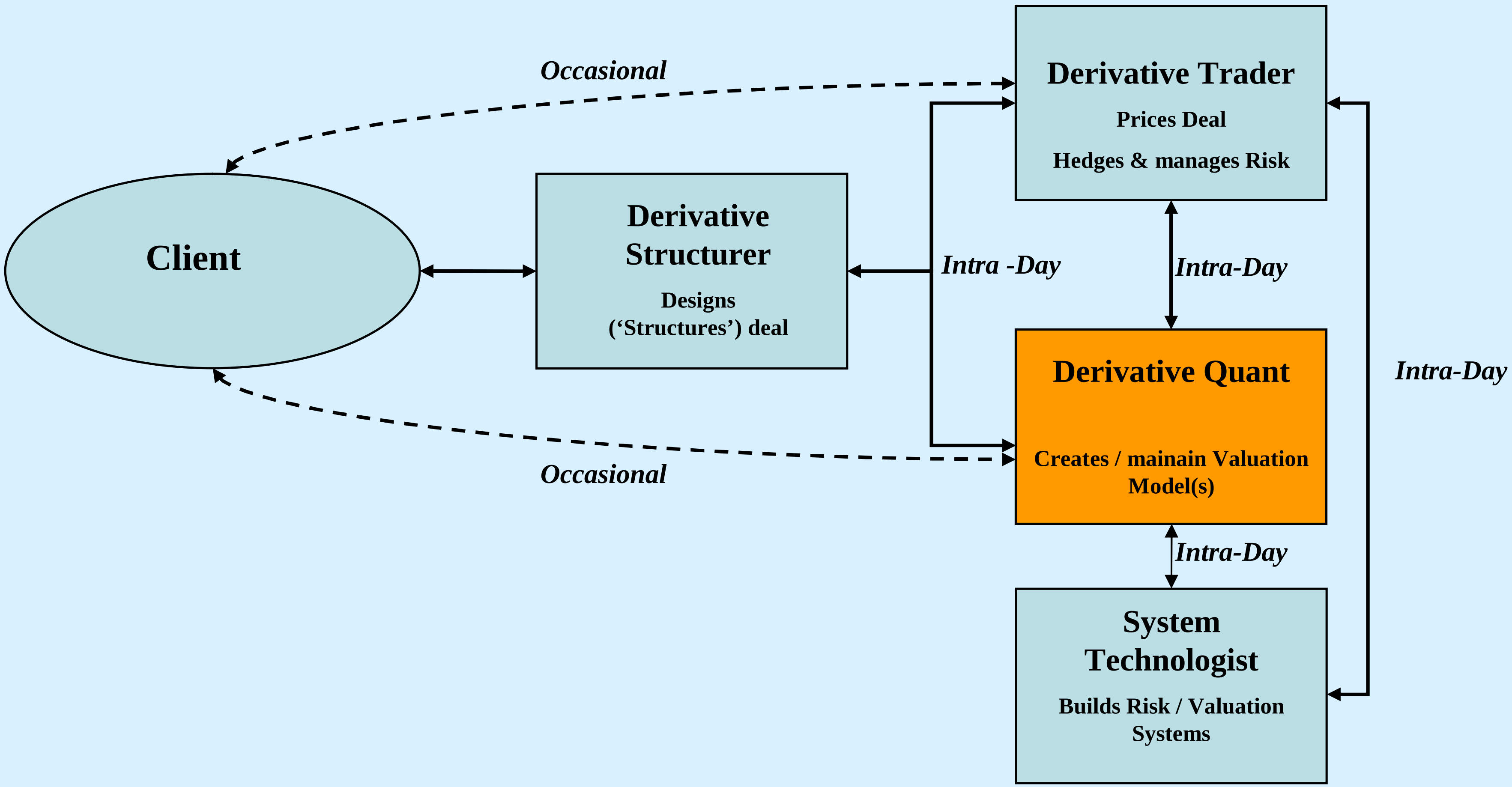


Typical Day for a Quant

- **6:00am-7:45am**
  - **Read FT and/or Financial research paper**
  - **Read Blackberry messages**
- **7:45-8:15 am**
  - **Arrive in office**
  - **Look at markets, read recent emails**
  - **Check status of any overnight computational batch jobs (usually calibrations)**
  - **Get breakfast and eat at desk**
  - ***(Tokyo market closing)***
- **8:15am – 7:00pm**
  - **Continue work on ‘long term’ modelling projects**
  - **Daily / weekly conference call with NY / Tokyo Quant teams (duration typically 1+ hour)**
  - **Write up any weekly/monthly research reports**
  - **Visit client(s) and give presentation on techniques of option valuation, answer questions and build / strengthen client relationship**
  - **Interview new quants**
  - **If quiet, go to corporate gym for ¾ hour**
  - **React to any client valuation enquiry**
    - **E.g. Client wants quote in $500,000,000 for a non-standard structured note by end of business today!**
  - ***(12noon – NY market opening)***
  - **Eat lunch at desk**
  - ***(6pm London market closing)***
  - **Set up any overnight batch jobs that have to be run**
  - **Continue to work on any (urgent) client pricing issues**
- **7pm-10pm**
  - **Leave office and go home**
  - **Dinner with clients / colleagues - either internally (in office) or externally (at restaurant)**
  - **Have any follow-up conversations with NY**
  - **Read Blackberry messages**
  - ***(10pm NY market closing)***

Typical Capital Markets Trading Floor - Canary Wharf

Daily Interactions of a Quant…



Essential attributes of a Quant

Mixture of technical skills and interpersonal skills…

- **Excellent mathematical modelling skills**
  - *Ph.D level in (Applied) Mathematics, Physics, Econometrics, Statistics, Finance*
- **Very good programming skills in C++**
- **Ability to deliver implementable solutions within time constraints**
- **Ability to work hard and remain focused and flexible under pressure**
- **Attention to detail, financial intuition / common sense**
  - *Mistakes cost money (literally)*
  - *Good, intuitive understanding of 'value'*
  - *Need to understand complex models and also 'back of the envelope' valuations*
- **Ability to work well as part of a (global) team**
- **Ability to work well on own**
- **Good communication / listening abilities**
- **Ability to 'reach out' and forge working relationships / partnerships across businesses / geographies**
- **Genuine interest in financial markets**
- **Energy, drive, determination, stamina, common-sense and realism**

Career Path of a Quant

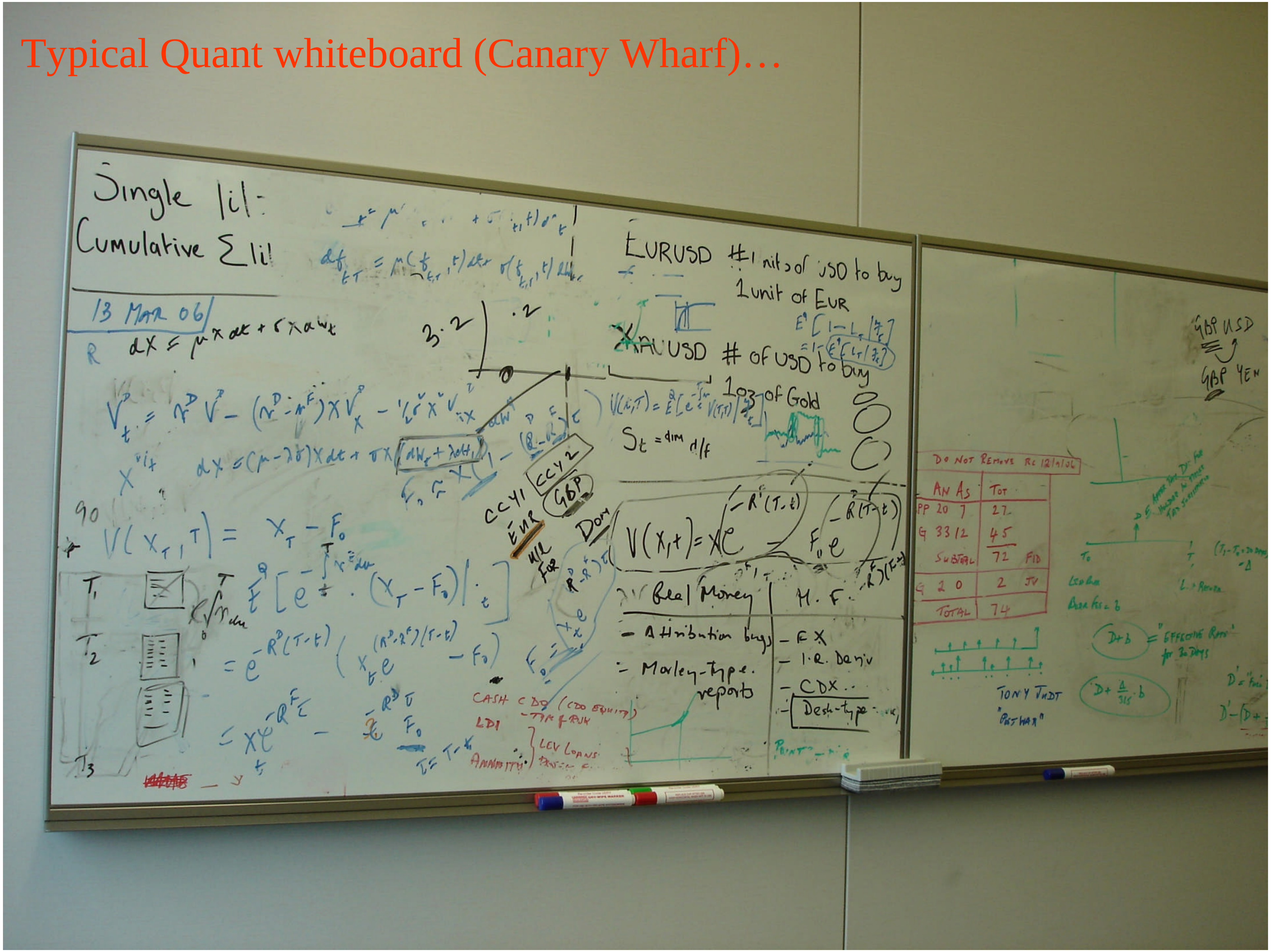
- **0-3 years**
  - Ph.D Associate
  - Work under close supervision
  - Learn products / modelling techniques / client base

- **3-6 years**
  - Vice President
  - Assume responsibility for products / models
  - Liaise / work with traders/structures on new product valuation and risk
  - Speak directly to junior clients
  - Manage 1-3 junior Quants

- **6-8 years**
  - Senior Vice President
  - Assume responsibility for a Quantitative group (e.g. Rates, Credit, FX, etc)
  - Responsible for new model development / maintenance of existing model
  - Speak directly to client Portfolio Managers (PMs) and Hedge Fund traders
  - Manage 3-10 Quants

- **8-12 years…**
  - Managing Director
  - Assume responsibility for whole / significant part of Quant Research organisation
  - Responsible for developing strategy for new model development
    - Products, headcount, technology, clients
  - Speak to senior clients (Chief Investment Officers, Heads of Fixed Income, etc)
  - Manage 10-100+ Quants and related staff

Typical Quant whiteboard (Canary Wharf)…



10Y CMS Swap

London: + 44 20 7102 4000
New York: +1 212 526 8163

Final Terms and Conditions

15 January 2007

Counterparties	
Party A	Lehman Brothers Special Financing Inc. ('LBSF')
Party B	**Client**
Notional Amount	**EUR 500,000.000**
Trade Date	15 January 2007
Effective Date	**31 January 2007**
Termination Date	**31 January 2017**
Party A Payments	**101% * 10Y EUR SWAP**
Payment Dates	**31 January in each year from and including 31 January 2008 to and including the Termination Date**
Basis	ACT/ACT ISMA
Period End Dates	Unadjusted
Business Day Convention	Following
Party B Payments	**1m Euribor +0.32%**
Payment Dates	**31st in each month in each year from and including 28 February 2007 to and including the Termination Date**
Basis	Act/360
Period End Dates	Adjusted
Business Day Convention	Modified Following
Definitions	**1m EURIBOR** : With respect to a Calculation Period, the rate for deposits in euros for a period of 1months which appears on Telerate Page 248 as of 11:00 a.m. Brussels time on the day that is two TARGET Settlement Days prior to the first day of such Calculation Period. **10Y EUR SWAP**: With respect to a Calculation Period the annual swap rate for euro swap transactions with a maturity of 10 years, which appears on the Reuters Screen ISDAFIX2 Page under the heading "EURIBOR Basis - EUR" and above the caption "11.00 AM C.E.T." as of 11.00 a.m., Frankfurt time, on the day that is two TARGET Settlement Days prior to the first day of such Calculation Period.
Business Days	London and TARGET
Calculation Agent	Party A
Documentation	All capitalised terms used in this termsheet and not otherwise defined will have the meanings given to them in the 2000 ISDA Definitions

Quant skills have many applications…

Appendix

What are Structured Notes?

- **Customised** financial instrument that **pays investor a series of pre-defined contingent cashflows at agreed dates in the future**

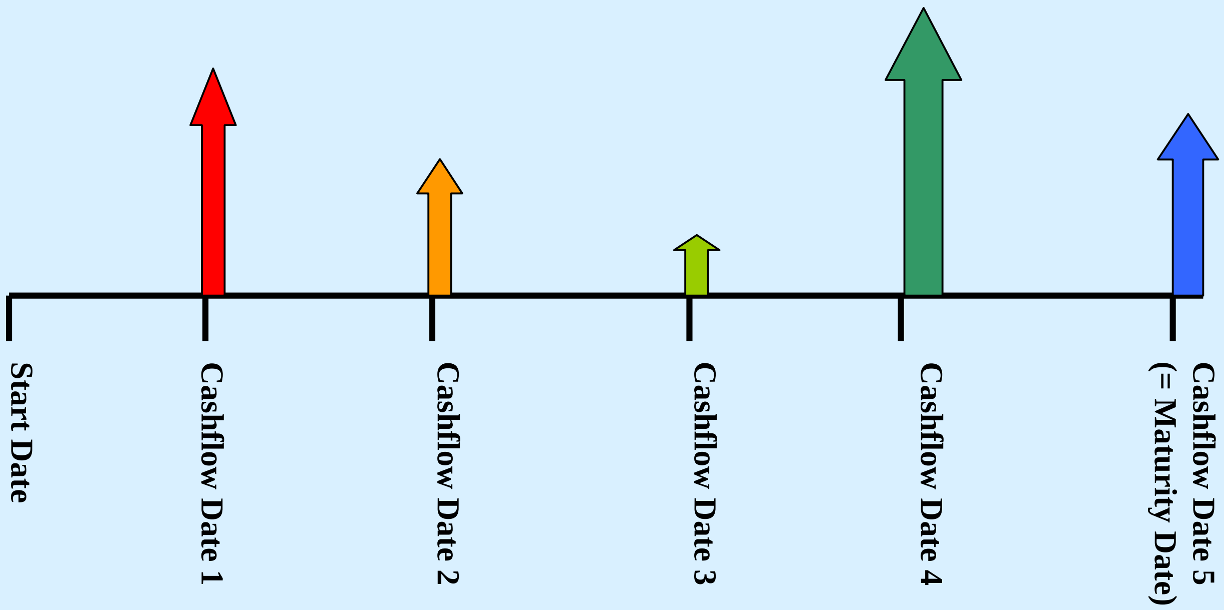
- Structured Notes enable
  - Borrowers to access cheaper funding
  - Investors to access bespoke cashflows that express and monetise their views on the market
  - Investment Bank to take a fee for arranging this deal
  - Everyone is a winner…

- Cashflows are **pre-defined function of underlying reference levels** (e.g. interest rates, FX rates, etc) in the future

- Model dynamical evolution of underlying reference levels (e.g. interest rates, FX rates, etc)

- Value using ‘delta-hedging / no-arbitrage’ techniques

Structure Notes - Cashflows



Cashflows are pre-defined function of (multiple) underlying reference levels (e.g. interest rates, FX rates, etc)

So how are Structured Notes actually created?

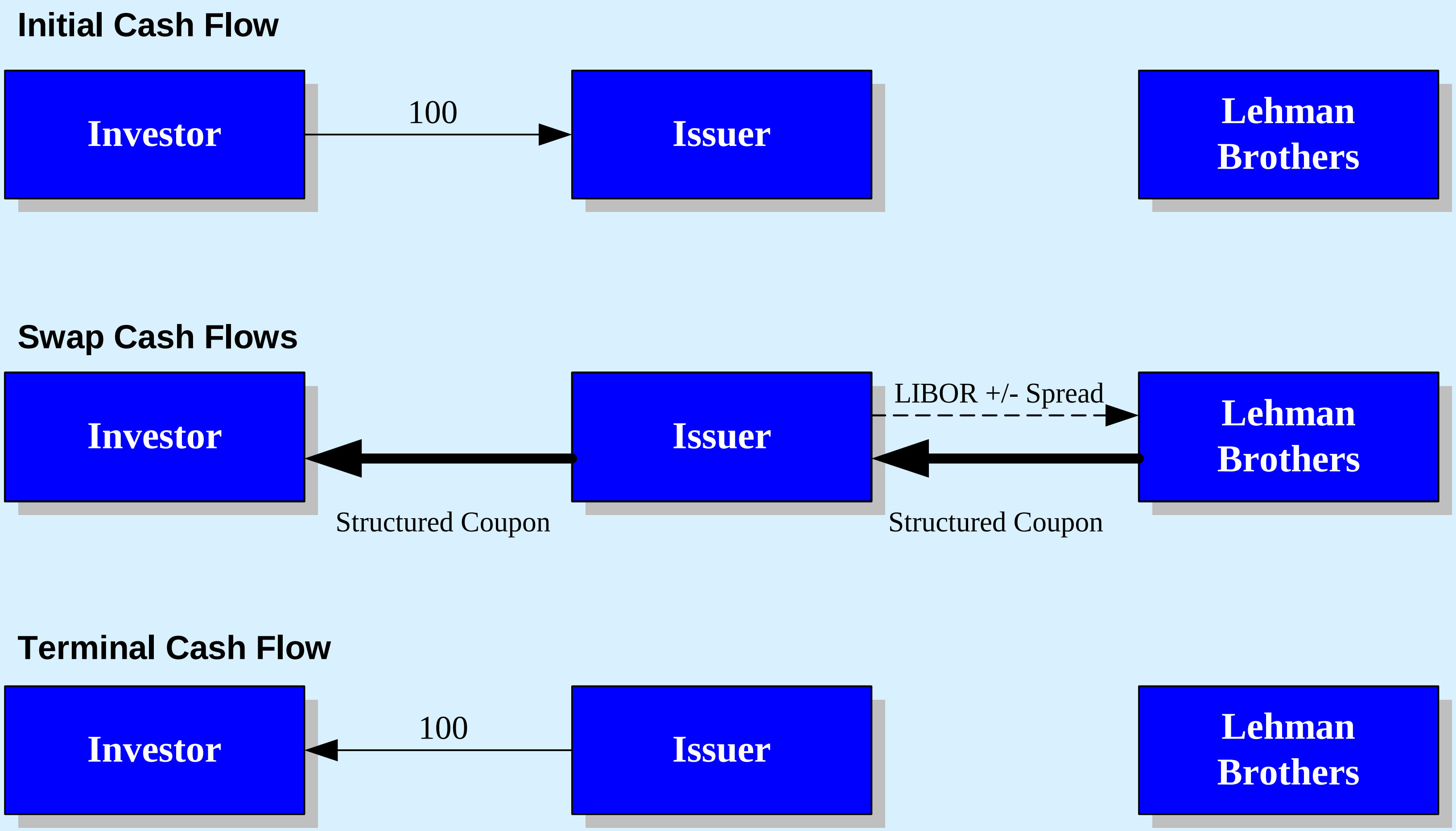
Example

- Suppose Issuer currently funds at LIBOR flat
- Suppose Issue would like cheaper ('sub LIBOR') funding

Method

- Find investor who wants to monetise a 'view' on market (e.g. changes in interest rate/volatility term structure)
- Lehman Brothers structures a coupon bearing note that encapsulates this view
- Issuers sells note to investor for PAR ('borrowing')
- Issuers enters into swap with Lehman Brothers
- Issuer repays investor PAR at maturity ('re paying')

Cashflow Structure for a Generic Structured Note



10Y Leveraged Callable CMS Steepener

100% Principal Protected

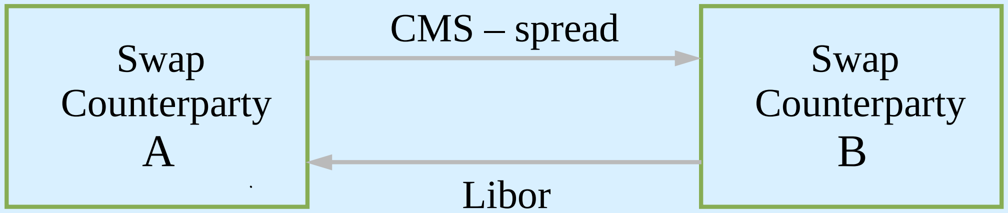
Final Terms and Conditions

9 February 2007

London: + 44 20 7102 4000
New York: + 1 212 526 8163

Issuer	Lehman Brothers Treasury Co. Bv
Nominal Amount	**EUR 250,000,000**
Trade Date	**9 February 2007**
Issue Date	**9 March 2007**
Maturity Date	**9 March 2017 (Subject to Issuer's Call Option)**
Issue Price	100%
Redemption Price	100%
Coupon Rate	**Year 1 – Year 4: 10Y EUR SWAP + 0.25%** **Year 5 – Year 10: 16* (10Y EUR SWAP – 2Y EUR SWAP) with a minimum coupon of 2% and a maximum coupon of 10%.**
Coupon Payment Dates	9 March in each year from and including 9 March 2008 to and including the Maturity Date
Calculation Period	From and including one Coupon Payment Date (or the Issue Date in respect of the first Calculation Period) to but excluding the next Coupon Payment Date each date being subject to no adjustment
Basis	ACT/ACT ISMA
Definitions	**2Y EUR SWAP**: With respect to a Calculation Period the annual swap rate for euro swap transactions with a maturity of 2 years, which appears on the Reuters Screen ISDAFIX2 Page under the heading "EURIBOR Basis - EUR" and above the caption "11.00 AM C.E.T." as of 11.00 a.m., Frankfurt time, on the day that is two TARGET Settlement Days prior to the first day of such Calculation Period. **10Y EUR SWAP**: With respect to a Calculation Period the annual swap rate for euro swap transactions with a maturity of 10 years, which appears on the Reuters Screen ISDAFIX2 Page under the heading "EURIBOR Basis - EUR" and above the caption "11.00 AM C.E.T." as of 11.00 a.m., Frankfurt time, on the day that is two TARGET Settlement Days prior to the first day of such Calculation Period. **Issuer's Call Option:** **The Issuer has the right on 9 March of every year starting 9 March 2011, provided that the Issuer gives 5 Business Days notice to the noteholders, to call the Notes at par.**
Business Days	London
Business Day Convention	Following

Constant Maturity Swaps (CMS Swaps)



$$\text{Spread} = \frac{1}{2} \cdot \frac{\Gamma_{FV01}}{\left|\partial_{FV01}\right|} \cdot \sigma^2 \cdot T^2$$

Exchange CMS – spread for Libor flat

- Spread chosen such that NPV of swap is zero
- Sensitive to slope of swap curve

Spread is a function of

- Steepness of the forward curve
- Volatility of the forward curve
- Maturity of deal

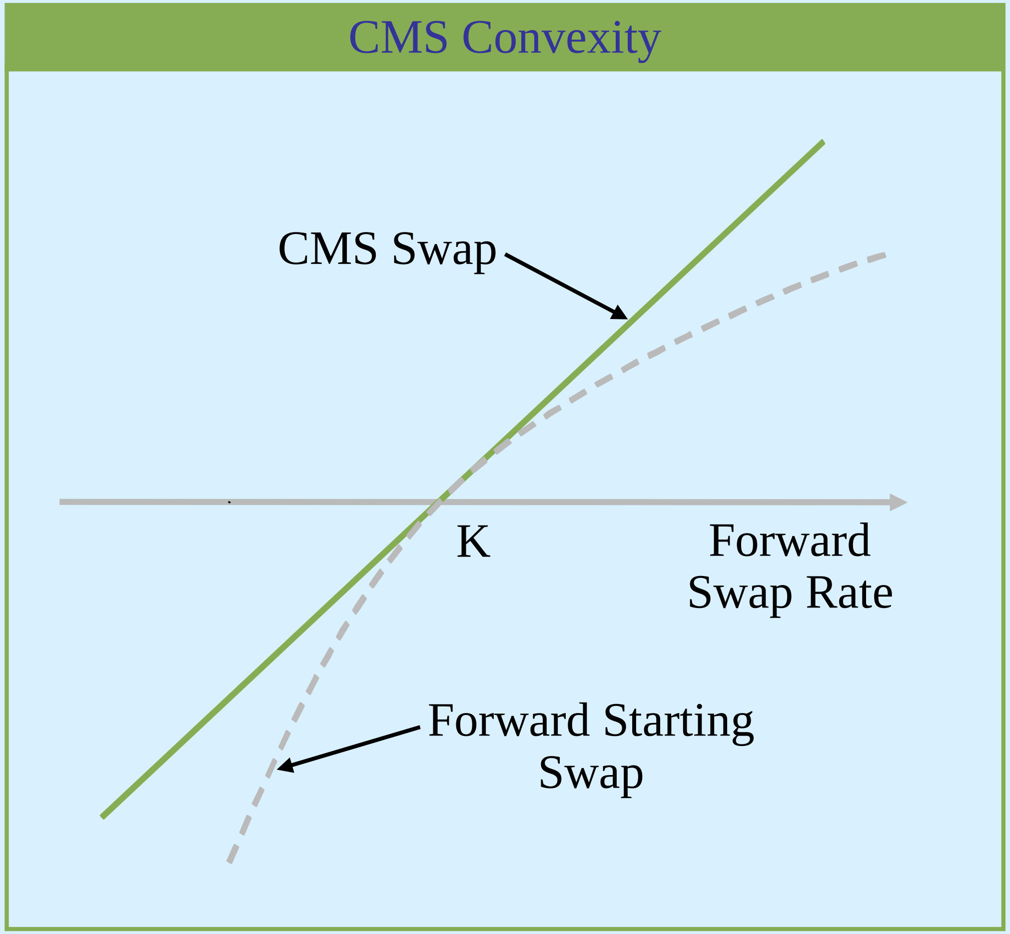
Receiving CMS / Paying LIBOR flat

- Long convexity/gamma/volatility
- Swap value increases with volatility

Paying CMS / Receiving LIBOR flat

- Short convexity/gamma/volatility
- Swap value decreases with volatility

Constant Maturity Swaps (cont'd)



Constant Maturity Swaps: Valuation – Step I

Example: 10Y Swap: Receive 10Y CMS – Spread / Pay LIBOR Flat

Compute how much the steepness is worth under a ‘zero volatility’ assumption

Today: 20-Mar-03 Spread: 0.0 bp

Acc Start & Fixing Date	Acc End & Pay Date	Fwd Swap Rate	Lognormal Volatility	CMS Cvx Corr (bp)	CMS Cpn	CMS Cpn – Spread	Fwd LIBOR	Discount Factor	PV CMS Leg	PV Float Leg
22-Mar-03	22-Mar-04	4.383	0.00	0.00	4.383	4.383	2.443	0.9756	4.276	–2.423
22-Mar-04	22-Mar-05	4.724	0.00	0.00	4.724	4.724	3.023	0.9466	4.472	–2.901
22-Mar-05	22-Mar-06	5.017	0.00	0.00	5.017	5.017	3.789	0.9116	4.574	–3.502
22-Mar-06	22-Mar-07	5.232	0.00	0.00	5.232	5.232	4.279	0.8737	4.571	–3.791
22-Mar-07	22-Mar-08	5.394	0.00	0.00	5.394	5.394	4.612	0.8346	4.502	–3.913
22-Mar-08	22-Mar-09	5.52	0.00	0.00	5.52	5.52	4.903	0.7950	4.388	–3.952
22-Mar-09	22-Mar-10	5.615	0.00	0.00	5.615	5.615	5.118	0.7558	4.244	–3.922
22-Mar-10	22-Mar-11	5.685	0.00	0.00	5.685	5.685	5.333	0.7170	4.076	–3.877
22-Mar-11	22-Mar-12	5.726	0.00	0.00	5.726	5.726	5.434	0.6795	3.891	–3.754
22-Mar-12	22-Mar-13	5.750	0.00	0.00	5.750	5.750	5.534	0.6434	3.700	–3.610
									42.694	–35.645

Important

- At zero spread and zero volatility, swap is worth 7.049 per 100 face
- As expected, value of fixed leg is greater than the value of floating leg
- This comes from the steepness of the EUR forward curve

Constant Maturity Swaps: Valuation – Step II

Example: 10Y Swap: Receive 10Y CMS – Spread / Pay LIBOR Flat

Compute the spread under ‘zero volatility’ assumption

Today: 20-Mar-03   Spread: 86.65 bp

Acc Start & Fixing Date	Acc End & Pay Date	Fwd Swap Rate	Lognormal Volatility	CMS Cvx Corr (bp)	CMS Cpn	CMS Cpn – Spread	Fwd LIBOR	Discount Factor	PV CMS Leg	PV Float Leg
22-Mar-03	22-Mar-04	4.383	0.00	0.00	4.383	3.516	2.443	0.9756	3.431	–2.423
22-Mar-04	22-Mar-05	4.724	0.00	0.00	4.724	3.857	3.023	0.9466	3.651	–2.901
22-Mar-05	22-Mar-06	5.017	0.00	0.00	5.017	4.151	3.789	0.9116	3.784	–3.502
22-Mar-06	22-Mar-07	5.232	0.00	0.00	5.232	4.365	4.279	0.8737	3.814	–3.791
22-Mar-07	22-Mar-08	5.394	0.00	0.00	5.394	4.528	4.612	0.8346	3.779	–3.913
22-Mar-08	22-Mar-09	5.52	0.00	0.00	5.520	4.653	4.903	0.7950	3.700	–3.952
22-Mar-09	22-Mar-10	5.615	0.00	0.00	5.615	4.748	5.118	0.7558	3.589	–3.922
22-Mar-10	22-Mar-11	5.685	0.00	0.00	5.685	4.818	5.333	0.7170	3.455	–3.877
22-Mar-11	22-Mar-12	5.726	0.00	0.00	5.726	4.860	5.434	0.6795	3.301	–3.754
22-Mar-12	22-Mar-13	5.750	0.00	0.00	5.750	4.884	5.534	0.6434	3.141	–3.610
									35.645	–35.645

Important

- Zero volatility spread is 86.65 bp
- This comes from the steepness of the EUR forward curve
- We expect that convexity effects will increase this spread

Constant Maturity Swaps: Valuation – Step III

Example: 10Y Swap: Receive 10Y CMS – Spread / Pay LIBOR Flat

Solve for the spread that makes swap NPV zero

Today: 20-Mar-03 Spread: 96.13 bp

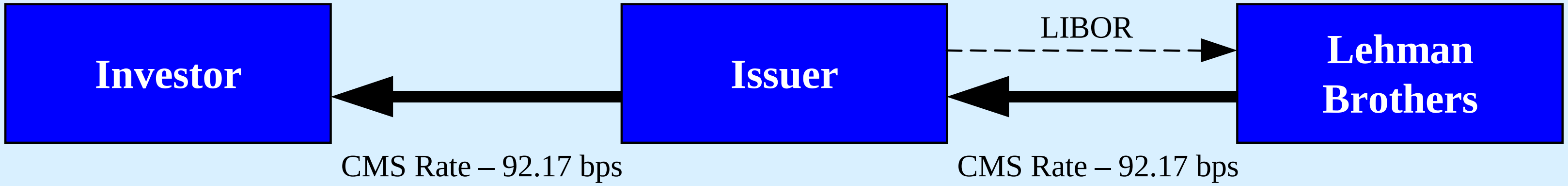
Acc Start & Fixing Date	Acc End & Pay Date	Fwd Swap Rate	Lognormal Volatility	CMS Cvx Corr (bp)	CMS Cpn	CMS Cpn – Spread	Fwd LIBOR	Discount Factor	PV CMS Leg	PV Float Leg
22-Mar-03	22-Mar-04	4.383	20.84	0.02	4.383	3.422	2.443	0.9756	3.338	–2.423
22-Mar-04	22-Mar-05	4.724	16.15	3.03	4.754	3.793	3.023	0.9466	3.59	–2.901
22-Mar-05	22-Mar-06	5.017	14.35	5.4	5.071	4.110	3.789	0.9116	3.747	–3.502
22-Mar-06	22-Mar-07	5.232	13.43	7.73	5.309	4.348	4.279	0.8737	3.799	–3.791
22-Mar-07	22-Mar-08	5.394	12.7	9.82	5.493	4.531	4.612	0.8346	3.782	–3.913
22-Mar-08	22-Mar-09	5.52	12.09	11.69	5.637	4.675	4.903	0.7950	3.717	–3.952
22-Mar-09	22-Mar-10	5.615	11.69	13.61	5.751	4.789	5.118	0.7558	3.620	–3.922
22-Mar-10	22-Mar-11	5.685	11.35	15.4	5.839	4.878	5.333	0.7170	3.497	–3.877
22-Mar-11	22-Mar-12	5.726	11.09	17.12	5.897	4.936	5.434	0.6795	3.354	–3.754
22-Mar-12	22-Mar-13	5.750	10.87	18.73	5.938	4.976	5.534	0.6434	3.201	–3.610
									35.645	–35.645

Important

- Convexity (volatility) increases the spread by 9.48bp from 86.65bp to 96.13 bp
- This is the average CMS convexity correction

Sub-LIBOR Funding – The Mechanics

Zero NPV:



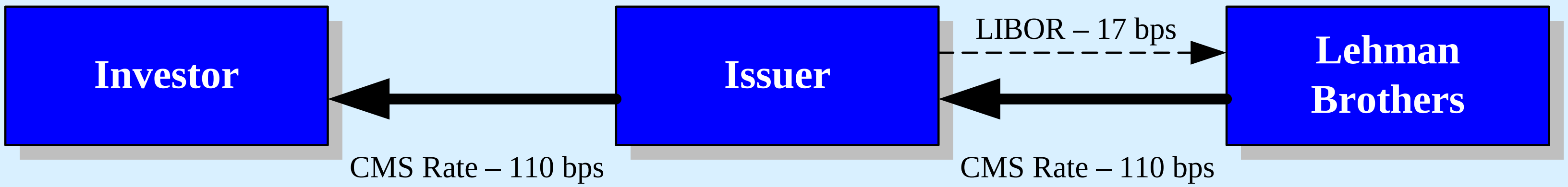
•Fair ('model') value = 98.60

•Issue sells note to Investor 100.00 (Par)

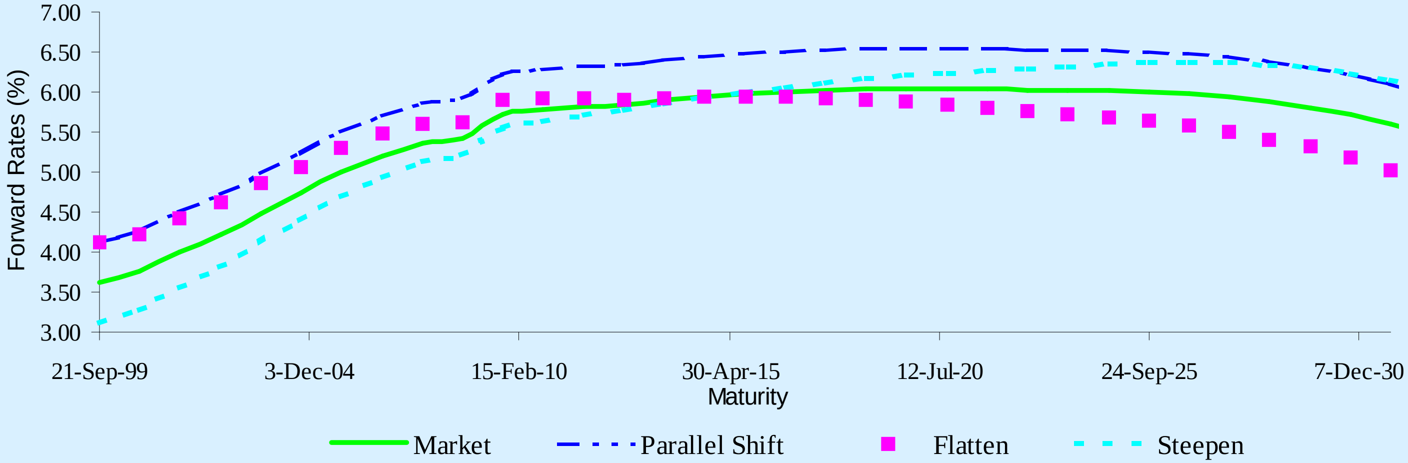
•1.4 difference used to subsidise cost of borrowing

•1.4 translates into 17bp year

Sub-LIBOR Financing:



Change in Value of a Constant Maturity Swap Under Different Curve Assumptions



Currently CMS is worth 100.00

1. **PARALLEL SHIFT of 50 bps, CMS value changes to 100.07**

2. **(30Yr – 3mth LIBOR) STEEPENING of 100bps, CMS value changes to 101.36**

3. **(30Yr – 3mth LIBOR) FLATTENING of 100 bps, CMS value changes to 98.71**